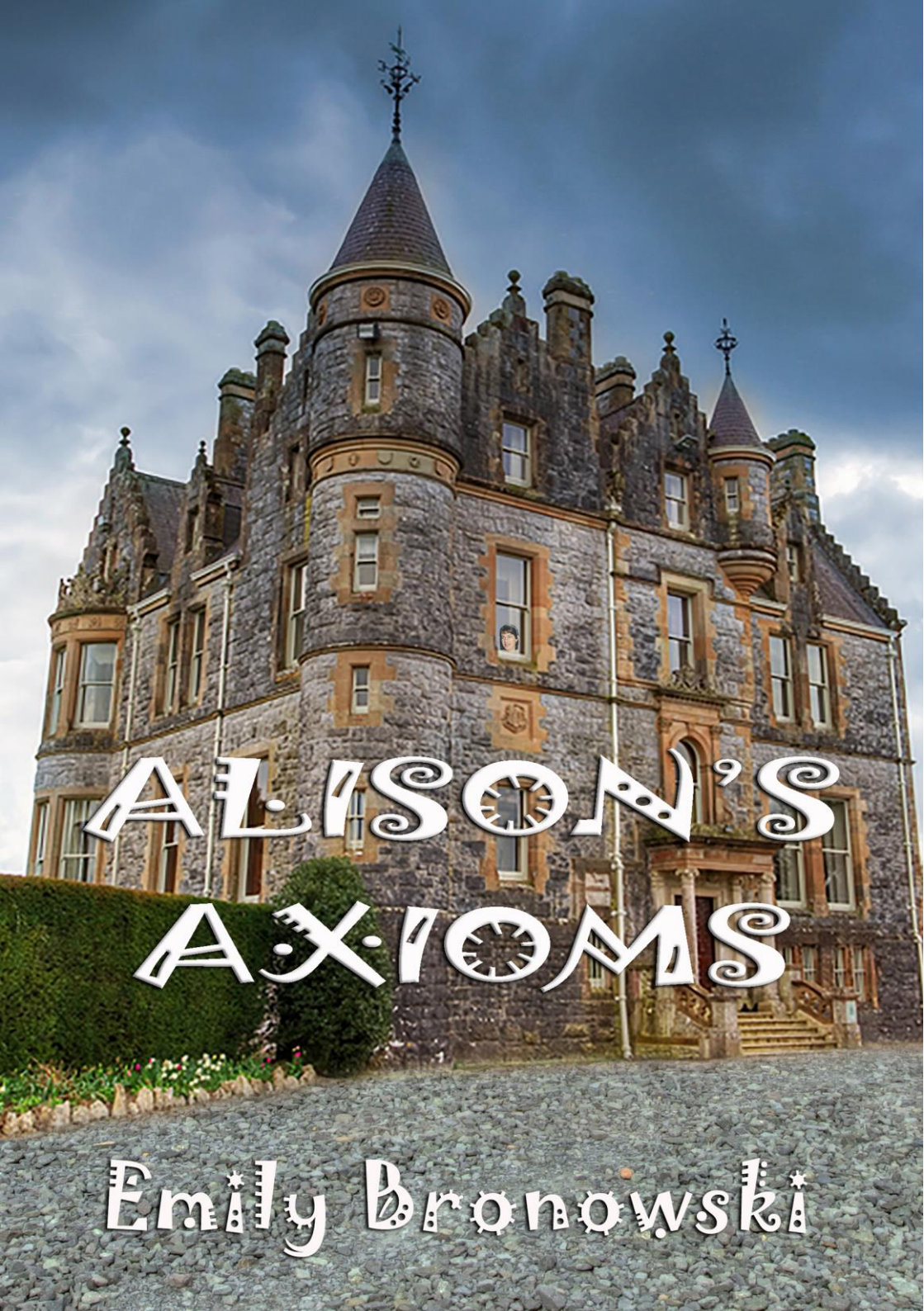




ALISON'S AXIOMS

Emily Bronowski

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READ THIS FIRST

So you are doing what I say! It's amazing how much power an author has over her readers. I suppose if I said DRINK THIS you would all drink from whatever bottle I gave you.

Anyway, if you are under ten years of age let me suggest that you skip this boring stuff and jump right into the story, feet first.

Now for the grownups, you might have guessed from the title of this book that it is inspired by Lewis Carroll's *Alice In Wonderland*. Certainly the intended readers of *Alison* are the same as those who appreciate *Alice*, that is, young children, adults who love whimsy and fantasy, scientifically minded people who enjoy the logic that lies behind the fantasy, and lastly, queens of England. It is said that Queen Victoria enjoyed *Alice* so much that she asked the author to dedicate his next book to her. She didn't realise that Lewis Carroll was the *nom de plume* of the mathematics lecturer, Charles Dodgson. So he did what she asked and dedicated *An Elementary Treatise on Determinants* to Her Majesty. See what trouble *noms de plume* cause. (Dodgson actually denied this story, but I'm not sure you can believe anything he said!)

So this book has been inspired by *Alice*, but in no way could it have been called *The Further Adventures of Alice*. It is different in several ways. For a start the action takes place, not in a Wonderland, but in real England and Scotland of some years ago. It is true that some strange things happen

there that might seem to be unbelievable, so you had better come to it in the same spirit of wonderment and belief in strange things that you did when you read *Alice*.

Secondly, Alice's adventures were episodic and the chapters could have been shuffled like a pack of cards, except for the first and the last. With *Alison* the adventures involve a quest, so the story progresses in a definite direction until the Ring is found. Oh, yes there's a Ring of Great Power in the story, just as in many other great quests.

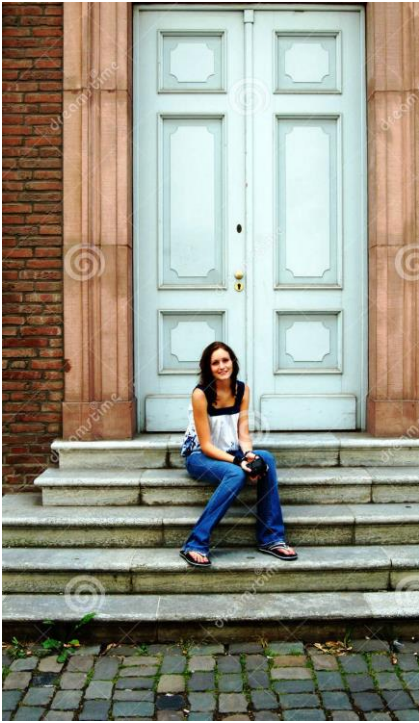
Finally there are almost no talking animals in this story. They didn't have talking animals in England at the time in which this is set. Oh, yes, there *is* Galois the talking beaver. You might think that I've borrowed Carroll's talking beaver from his wonderful poem, *The Hunting Of The Snark*, but no – Galois is from French Canada and is much, much braver.

I expect you knew this already because, if you are anything like me, you will have skipped this Preface and waited till you had read the rest of the book before returning to it. But now, let's try to catch up with the younger readers and step gently into chapter one.

THE MÖBIUS LINE

Alison was beginning to get very tired of sitting, waiting for her sister on the front steps of number 24 Leinster Gardens, London WC1. The water kept dripping off the edges of her umbrella and she was feeling quite miserable.

“Just like Emily to keep me waiting,” she thought. “We agreed to meet here at four o’clock and it’s already half past.”



Emily was her younger sister and she was not at all like Alison. She believed that she was the reincarnation of Emily Brontë because she had a logical mind, and liked animals so much better than people – and she especially loved the Yorkshire moors. But, to tell the truth, she had only been to Yorkshire once and that was in the winter and, although she *did* go for a walk across the moors she hadn’t gone very far before the cold winds forced

her back. Still, she knew that if she *had* lived in Haworth she would have loved the moors as much as Emily.

She was christened “Emily,” but ever since fourth class at school she had spelt it ‘Emily’. She thought the two dots over the “e” made her quite special. Emily Brontë had the

two dots over the “e” in ‘Brontë’. But, with the name Emily Lucinda Christianson, she had no other “e” she could coronate. So Emily she became, even though her sister Alison thought it was rather silly to have the two dots over a capital letter.

Now Alison was also very interested in special characters, but she felt very strongly that they needed to be used properly. She had explored the symbols section on her father’s computer and she was always asking him what they meant and why they were shaped the way they were.

She loved the word “encyclopædia” because of the way the ‘a’ and the ‘e’ were combined as if they had been squashed between the ‘p’ and the ‘d’. And she was fascinated by the fact that whenever she typed ‘naïve’, the computer insisted on putting two dots over the ‘i’ instead of one.

Of course you may never have noticed this because you may have had very little occasion to use the word. Alison frequently used it when she was typing up her diary. She often recorded that Emily was rather naïve for one reason or another.

But Alison’s favourite was the cedilla. This is a funny little squiggle that goes under the letter ‘c’. It is used a lot in French. When the word ‘façade’ emigrated to England it continued to speak with a French accent.

Alison stumbled on the cedilla. When she typed the word ‘facade’ on her father’s computer it insisted on putting the little squiggle under the ‘c’. Emily said she didn’t believe there was such a word, but Alison said that it meant the front of a building.

“What other words have a silly dilla?” asked Emily.

“I can’t think of any just at the moment.”

“Just as I thought. You’re making it up!”

But Alison was not making it up. There *is* such a word as ‘façade’ and it should always come with the little seahorse clinging to the letter ‘c’. “It’s a seahorse under the sea,” she used to say to Emily but Emily didn’t think that was a very funny joke.

Thoughts such as these were strolling through Alison’s mind as she was getting very wet because of the steady rain. And she was starting to get very bored.

“Perhaps a white rabbit will run past,” she thought just to keep herself amused. She had always been teased about her name. “Pardon me, Miss Wonderland, where have you left your white rabbit?” she would be asked. Once she even got a rather rude letter sent to her, addressed as ‘Alison Wonderland’.

In sixth class she had contemplated changing her name, like Emily. She wanted to spell it “Aliçon”. She reasoned that those ignorant people who would make fun of her name wouldn’t know how to pronounce it and she would be spared their constant teasing. But, in the end, she left it as it was.

Just then a white rabbit came hurrying up, consulting a large pocket watch. Well, actually he wasn’t *really* a rabbit. If you think that this is one of those fairy stories, full of talking animals, you are very much mistaken. No, something cool and solid lies before you – something as ordinary as Monday morning when you have to go off to school. If you like reading such fanciful stories you might be better off with a different book! I can suggest some very good ones. But, if

you insist on continuing to read *this* one I have to warn you that you'll meet very few talking animals.

So the creature who was in such a hurry was no rabbit. He was a funny little middle-aged man wearing a white cassock – the sort of robe that monks sometimes wear. I can see why you might have thought that he was a white rabbit, such as the one Alice met, because he *did* have whiskers and his nose twitched in a rabbit sort of way. But if you look closely you'll see that *this* rabbit was wearing sandals, which proves that he wasn't a rabbit after all. If you are going to make sense of this story you'll have to pay attention to detail!

"Excuse me young lady, I need to get past. You see I'm quite late," he said as he consulted the gold watch he had taken out of his pocket.

"Oh no, you won't do any good going into this house," Alison replied helpfully. "You see it's a fake house – it's just what they call a façade."

Alison loved to use the word. It was such a pretty word to pronounce. But unfortunately it doesn't often come up in conversation. Sometimes she steered the conversation so that she could use it, though often she used it somewhat incorrectly.

"I love the façades of that book," she might say, or "take that grin off your façade". But here was an opportunity to use the word correctly. She went on to explain.

"No, you see, the Circle Line runs under our feet, so behind that wall it's just an open hole with a train line

running along the bottom. When they built these terraces they didn't want to leave a gap so they made a false façade."

"See, look at those windows," she said pointing to the upper floors. There's no glass in them – they're simply painted on the wall."

"That's very interesting," he said as he pushed past her, "what will they think of next?" Then he walked up to the false front door, turned the real brass handle, opened the fake door and walked inside!

Alison jumped up in surprise. She had lived in this street for several months and she had always known this house to be merely a façade. She had even gone around to the street behind and seen that the building had no substance. But the funny little man had gone inside, and so she ran after him and got inside before the door slammed shut.

I know what you're thinking. On the other side of that door that wasn't supposed to open, was a hole with a railway track below it and that Alison was in danger of falling. She didn't look to see what was on the other side but luckily, instead of being an empty hole, open to the sky as she had always believed, she found herself in a large station concourse, with a row of old-fashioned ticket windows along one side. They were shaped like church windows, with a half circle at the top, and they had shiny brass bars to stop people jumping through the window and stealing all the money, and roller blinds with red and white diagonal stripes that could be pulled down when the window was closed.

Suddenly, from high above her head, Alison heard a peal of bells.

ding
ding
ding
ding
ding
ding
dong!

She looked up. So they were ringing rounds on eight bells – nothing very complicated. She and her father were bell-ringers at Christ Church, Lancaster Gate.

But where were these ringers? Then, as she looked back at the ticket windows, she noticed that the roller blinds were going up and down. And as the ticket sellers behind the windows pulled their blind down, their bell, far above, rang out.

The ringers were bobbing up and down, though not in the technical sense of ‘bob’ that Alison had learnt in the bell-tower. Suddenly one of the ticket sellers was lifted off his feet and disappeared as he was wrapped around the blind on its roller.

“Stand!” came a loud voice and the ticket sellers brought their bells to rest. Just then, as one of them turned sideways, Alison noticed that he was two-dimensional. He had absolutely no thickness!

That’s what comes of being two-dimensional, she thought, you don’t have enough weight to control your bell. The two-dimensional ringers went to the aid of their team

member who was uncomfortably curled up in his blind. They pulled down the blind and a two-dimensional man fell out.

Something similar had happened in Alison's tower. One of the stays had broken, far up above, and the bell kept going round and round on its wheel. The ringer had been pulled up towards the ceiling. But, being three dimensional, he wasn't wrapped around the wheel, and in any case he had the presence of mind to let go. But it was scary.

Just then she remembered her 'white rabbit'. She wanted to catch up with him to ask him some questions about her strange experiences so she hurried through the turnstile, down the long flight of stairs and onto the platform.

He was at the far end of the platform and just then a train pulled in. Alison ran to the front carriage and jumped on, leaving the door wide open.

"Would the young lady in the front carriage please shut the door," called out the guard. "D'you think it'll close by itself?"

Alison, of course, had only been used to automatic doors, but this one had to be swung shut. She slammed the door, spotted the 'white rabbit' and sat down next to him as the train slowly puffed out of the station.

"This is a very old train," she said to the 'white rabbit'. "I didn't know they still had steam trains on the Circle Line.

"My dear, why do you think that house in Leinster Gardens is only a façade. The open space behind is to let the smoke out."

"But it wasn't a façade! There were ticket offices and everything. Don't you remember?"

“Why you told me not so long ago that it was a façade, don’t you remember? Of course it is a lady’s privilege to change her mind. Won’t you sit down?” So she sat down in the seat next to him, wondering what to believe. This afternoon had turned out to be rather strange.

Why had she followed him? She wasn’t quite sure. She knew that, by now, Emily would probably have come to their meeting place outside Number 24 Leinster Gardens and she would be saying things like “Just like Alison to be late.” But there was no turning back. Besides she felt that this was going to be an adventure. Emily could jolly well wait!

“Now, young lady,” “what were you telling me about façades?”

“Yes sir, except it wasn’t just a façade after all. It was a *real* building. I’ve lived in this street for four months and I always thought it was just a pretend house.”

“Things aren’t always what they seem ... but sometimes they are. I know it’s been twenty years since I was last in London and things can change after twenty years. I remember you telling me that it was just a façade but I believe in checking things out for myself. You see believing something is true just because someone tells you very earnestly that it is, can be misleading. It’s always a good idea to test things out for yourself, isn’t that so?”

Alison had recovered her breath and had begun to look around. She had travelled on the Circle Line many times but she had never seen a carriage so delightfully old-fashioned. Above the seats were brass racks to hold your luggage and

down the centre of the carriage roof there was a row of the cutest little lights you have ever seen, each shaped like a rose.

“I might get back in time for vespers after all,” said the ‘white rabbit’ pulling out his gold pocket watch from his waistcoat.

I keep calling him the ‘white rabbit’, but of course he was no such thing. It’s just that he had whiskers, and a white robe, and a nose that twitched. And he had a gold pocket watch. But, as I said, he wore sandals and I expect you’ve never seen a rabbit wearing sandals. And he had the funniest sort of haircut – bald in the middle.

“Are you a stranger,” asked Alison. “It’s just that I’m not supposed to talk to strangers.”

“Do *you* think I’m strange?” he asked her, his nose twitching harder than ever.

“Oh yes sir, if you don’t mind me saying so. You’re the strangest person I’ve ever seen.”

The white rabbit smiled. “Well then, you see, I’m not a stranger – I’m the *strangest*. Did your mother ever tell you not to talk to strangers?”

“No, I don’t think so, but I’m still not sure whether I should talk to you,” replied Alison.

“Well that’s because we haven’t been introduced. Once we’re properly introduced we can no longer be strangers, would you agree?”

On the seat behind was an old lady in a black straw hat, with a yellow rose – the sort of hat you sometimes used to see on a horse in the olden days. The ‘white rabbit’ turned around to this lady and asked, “would you do me the honour to introduce me to this young lady?”

“Certainly,” she said standing up and bowing, “Brother Charles I would like you to meet Alison Christianson. Alison Christianson I have the honour of presenting to you Brother Charles.”

“Delighted I’m sure,” said Brother Charles shaking Alison’s hand. “Now we are old friends.”

“But how did she know our names?” Alison asked screwing up her nose in bewilderment.

Brother Charles ignored such a silly question, but he was careful not to let on that he thought it a silly question. He was a kind old gentleman and, living in a monastery as he did, he prayed every night for God to make him gracious and kind to others.

Now I know that a lot of grown-ups will criticise this book because they will say that I am encouraging children to talk to strangers. In fact they will probably not allow it to be in school libraries for that very reason. The fact that *you* are reading this book now probably means that you have an enlightened school librarian, or perhaps a well-informed parent who has bought you your own copy.

Of course you should *not* talk to strangers, especially on a train. Some children have been kidnapped through talking to strangers and terrible things have happened to them. But you also know that strangers are not bad simply because they are strange, or because you don’t know them. Nearly all strangers are good people, if only you get to know them. But because there are a few bad strangers in this world it is best to be cautious. Anyway I shouldn’t be telling you

about strangers. That is what parents are for. You listen to them and obey them.

But the odd thing is that in stories children often talk to strangers. You have to remember that when you stumble into a different world, by walking through a magic door or falling down a rabbit hole, *all* the people you meet are strangers. If Alice hadn't talked to strangers she wouldn't have been able to talk to anyone in Wonderland and that would not have suited her at all, because she loved to talk. In fact you may have noticed that if there was nobody else around she talked to you, the reader.

So if ever you find yourself in a magic world, like you read about in books, then my advice is that there is nothing at all wrong in talking to strangers. But while you remain in *this* world, where you have parents and brothers and sisters and aunts and cousins and teachers and best friends then you have so many people to talk to you don't need to talk to strangers.

Anyway, if your parents say that you should not be reading about Alison talking to a stranger, just tell them that this book is a 'classic'. They will understand.

"So you're a monk," she asked, "are you sure that you aren't an abbot – the white abbot?" At this she grinned because she thought it a very funny remark.

If Brother Charles didn't think it all that funny (Emily would have thought it stupid) he didn't let on. Instead he smiled warmly and said, "I see you have read *Alice's Adventures Underground*."

"Oh yes, I think it the most curious book I have ever read, and I love curious things. For example I find this train

most unusual. I didn't know that London Transport used old carriages like this on the Underground."

"So your name is Alison. How curious to meet Alison on the Underground."

Alison winced – yet another poor pun on her name. But Brother Charles' face seemed so friendly that he couldn't possibly be trying to make fun of her.

There was a long silence and Alison could hear the rattling of the wheels on the track and the roar of the wind in the tunnel. Finally she broke the silence by remarking "That's a really old pocket watch you've got there? It must be over a hundred years old."

"This, yes I suppose it is. My uncle gave it to me when I entered the monastery in the early 90's. We aren't normally allowed to have valuable personal items at St Huberts – vow of poverty and all that – but the Abbot made a special exception in my case.

Alison showed him *her* watch. "I got this watch for my 16th birthday recently. It's a digital and the time and date scroll across the face continuously."

Brother Charles looked at it with great interest. "My, this is amazing. You see I come out into the world so rarely that each time I do I'm amazed at how the world has changed. We lead a very sheltered life at St Huberts. Every day it's prayer and meditation and mathematics."

"You do mathematics at the monastery? Is it like a school?"

"Oh no, it's my pursuit. We each of us have to undertake some pursuit. Some of the brothers do gardening,

others make furniture and one of the brothers studies herbs and runs the infirmary. Brother Bernard goes fishing, and supplies us with copious quantities of fish. Me, I do mathematics to the glory of God.”

“Don’t you find it boring? Emily loves mathematics, but I find it rather perplexing.”

“Oh no, some of the deepest mysteries in life can be found in mathematics. One can delve so deeply that one almost glimpses the very face of the Almighty! But what’s this?” he asked as he looked closely at the digital watch. It has the right time and the correct date but it says 2006.”

“But it is 2006 – isn’t it?” asked Alison beginning to doubt her memory.

“Good gracious child it’s only 1933. That watch is over seventy years fast!”

“But you said you entered the monastery in the 90s.”

“Yes, indeed – the 1890s – about forty years ago.”

It began to dawn on Alison what had happened. She had fallen down a deep hole. She had walked through the back of a wardrobe. When she entered that red door in the façade she had crossed over into another world, a world where time runs differently to the world she was used to.

But she was not in Wonderland, or Oz, or Narnia. She was in dreary old London on the Circle Line. Yet it was London in 1933!

Now you and I would be frightened at discovering we were in an entirely different world. In a different world you can’t trust your own judgement built up from experience. Why, anything can happen in a different world. But Alison

was not the least bit scared. What's that you say? You wouldn't be scared either. Well perhaps that's because you've lived less than a dozen years in this world. Let me tell you from my own experience that when you've lived a long time in one world it's a very frightening thing to stumble into a different one.

"These carriages are pretty fancy," said Alison to Brother Charles. "In the London of my day they're a lot more functional – more ads and fewer seats."

"They were even more special when the Circle Line opened in 1884. I remember my grandfather telling me about it. He said that the anti-clockwise trains were grander than the clockwise ones."

"But don't the same trains run in both directions?"

"And where do you think they would turn round on the Circle Line?"

"But I thought ..."

"Anyway," interrupted Brother Charles. "Anyway, the two different directions were run by two different companies. It was a joint project, but the Metropolitan Railway was responsible for the anti-clockwise trains and the District looked after the clockwise ones."

All this time the train had been hurtling along. The other passengers seemed to be unaware that they were in a different world. Some read newspapers, others dozed. Not one of them was speaking into a mobile phone or listening to music with earphones. But then this was 1933.

They had passed quite a number of stations but didn't stop at any of them.

"I thought the Circle Line was supposed to stop at every station," said Alison.

"But my dear, you must be on the wrong train. This is the Möbius Line, not the Circle Line. Never mind. Like the Circle Line the Möbius Line goes around in a loop. You'll soon be back where you started."

Just then a station was whizzing past, but upside down. As she looked up she could see the platform and the people waiting were waiting *under* the platform, not *on* it. "Stranger and strangest," thought Alison.

Brother Charles could see that Alison was puzzled so he thought he'd try to explain. "The Möbius Band is topologically different to the cylinder." Alison was still just as puzzled. "Who plays in the Möbius Band?"

Brother Charles chuckled. "No my dear, the Möbius Band is not a brass band like the one that plays in the rotunda in Regent's Park on Sunday afternoons." Alison had in mind more of a rock band like Midnight Oil.

Brother Charles turned round to the lady with the black straw hat and said, "may I borrow your hat for a moment?"

"Of course you can," she replied, taking off her hat.

He carefully removed the hat band and showed it to Alison. "*That* is a cylinder."

"I thought a cylinder was more like a can of soup."

"As I said we lead a very sheltered life in St Huberts. We make our own soup in large saucepans. Perhaps you mean that a saucepan is a cylinder. Well, I suppose it would be if you took off the handle and cut the bottom out. In fact

we do have a saucepan that's very like a cylinder. Brother Timothy was cooking a stew one night and the handle fell off."

"It was too hot to take it off the stove so he went to get some help, and he got distracted and forgot all about it. When it was discovered, all that was left of the saucepan was a cylinder, open at both ends, with a dark charred mess all over the stove. Well this hat band is a cylinder that happens to be not very tall. But it encompasses quite a large diameter."

"I think I see what you mean," said Alison hesitantly.

"Well, if we cut the band and give it a half twist and then join it up it becomes a Möbius Band." He cut the band, gave it a half twist, and then took out needle and thread from under his robes and in a moment he had neatly stitched the ends together.

"See it's only got one side. When you go around and get back to where you started you are upside down."

"My poor 'at band," cried the little old lady, "you done ruined it."

But in a trice (I think that's one third of a moment) he had cut the threads, untwisted the band and stitched it up again as neatly as the Tailor of Gloucester and presented it back to the lady.

After passing quite a number of stations, all of them upside down, the train came to a halt. Luckily this station was right way up. If you have ever tried getting out of a train *underneath* a station that is upside down you will know how difficult it can be. And what do you know? It was the very station that they had got on at.

I must apologise for ending a sentence with, not just one preposition, but two! But I've tried to rework it and it just doesn't work. It was the very station at which they had got on? It was the very station at which they had on ... No, it will have to stand.

"This is my station," said Brother Charles as he opened the carriage door. "I suppose it must be my station too," said Alison. "All change!" called out the guard. It was everybody's station.

I know what you're going to say. Since Brother Charles had got *on* at this station and had got *off* at the same station, it was all a waste of time. But that is because, like most people these days, you are too impatient, always wanting to get somewhere different. Haven't you ever had a ride on a merry-go-round at a fair and got off exactly where you got on? Did you *really* feel it was all a waste of time?

The old lady with the black straw hat also got off and stood with Alison and Brother Charles on the platform, waiting for another train.

"For which train are you waiting," Alison asked the old lady. She was careful not to end her sentence with a preposition.

"Excuse me young lady – I don't believe we've been introduced," said the old lady, looking down her nose as if Alison was a nobody.

She didn't look like a Duchess, but the haughty we she was now speaking made Alison wonder if she really was a Duchess in disguise. So she asked Brother Charles to introduce her.

“I’m sorry I can’t do that because I don’t know her,” he replied.

“But you asked that lady to introduce us so she must know you,” Alison protested.

“The knowing relation is not symmetric,” Brother Charles explained. “I may know you but that doesn’t mean that you know me. For example I know the bishop. I’ve met him on several occasions when he comes to visit St Huberts. But each time he sees me he asks, ‘do I know you?’. And it’s not transitive either. I know you and you know your sister, but that doesn’t mean that I know your sister.”

“No, of course not. I can see that. I wonder if she’s still waiting for me. I think knowing is a pretty hopeless relation.”

“Oh, it’s not all that hopeless. At least it’s reflexive. Everyone knows themselves, or at least they should.”

“Well what am I to do?” asked Alison.

“Well this lady clearly knows herself – that’s reflexivity – so let her introduce herself to you.”

“I’d be delighted to,” said to the lady with the black straw hat, giving a low curtsy. “Ivy Dunn is my name and you are Alison Lucinda Christianson. Pleased to meet you, I’m sure.” Alison still wondered how the old lady knew her name.

So from now on I can refer to ‘the lady with the black straw hat’ by the name ‘Ivy’, which is a little easier to type. Of course she may catch a different train and we may never see her again, in which case it wouldn’t really matter.

“That’s a nice name,” said Alison. She didn’t really like the name but she wanted to say something friendly.

“Oh, I’m sick o’ it. When I was a wee mite the other childer in the village used to call me ‘Drip’ or ‘Intravenous’. My teacher used to call me ‘4’ because ...

“Yes, I know – IV – Roman numerals.”

“Even now I get people askin’ if I’m climbing up the wall. Pretty potty if you asks me.”

“Oh, I’m sorry. Perhaps I should call you Mrs Dunn.”

“Most people where I works calls me Dunn, or Dunny.”

Alison giggled. Being from Australia the word ‘dunny’ was not something you called someone if you wanted to be polite.

“I’ll just call you Mrs Dunn.”

“Now what was it you wished to ask me my young lady?” Mrs Dunn asked Alison.

Alison curdled a little at the way she was addressed. “Now that you know my name there’s no need to call me *your* young lady.

“No offence deary ... I mean Miss Christianson ... I meant no ‘arm. It’s just the way we speaks where I comes from. You was askin’?”

Alison’s face brightened and her temper sweetened at this most gracious apology. “I was just wondering which train you were catching.”

“Oh, the District Line Miss Christianson. I live out in the District and I just came up to London to visit my sister in Bayswater. What about you?”

Now by rights, Alison should have remembered her sister Emily who surely by now would have been waiting for her for nearly an hour and she had no business to be

contemplating yet another journey. But she had realised that she was in another world, and she knew that when you were in another world you could spend as long as you liked and when you returned to your own world you wouldn't have been away for any time at all.

Alison remembered reading about Lucy who spent several weeks in Narnia and when Lucy eventually went back through the wardrobe her brothers and sister were still playing the same game of hide-and-seek that they had been playing when she had hidden herself in the old wardrobe.

"Why I'm waiting for the District Line too," said Alison. "The District sounds a lovely place, just like the Shire that the hobbits keep talking about."

"Oh, no we 'av no rabbits in the District. There used to be a plague of 'em, but not these days."

"What about you Brother Charles?" asked Alison.

"Yes, I too am awaiting the District Line train. You see our order is not a strict one and so our monastery is built in a pleasant place at the end of the District Line. Some of the stricter orders occupy monasteries in the far north where it always seems to be snowing."

"And never gets to Christmas," giggled Alison, but the other two didn't seem to know what she was referring ... to what she was referring. Yes, you're right. Coming from 1933 they could never have read *The Lion, the Witch and the Wardrobe*. Just a tip for the next time you find yourself back at an earlier time. Be aware that there are things that *you* know that the people you meet don't because it hasn't happened yet. If Alison had mentioned the disaster of 911,

Brother Charles would have wondered what was so bad about a number being prime.

As he said this, a train came rumbling into the station, and a funny man in a dark blue cap called out “Upminster train, District Line.”

THE DISTRICT LINE

The District Line carriage was even older and quainter than the one before. It had curtains on the windows, and little water bottles in racks on the wall, together with drinking glasses. The carriage was quite full. After they sat down every seat was occupied.

“Do you have any family or did you have to give them up when you entered the monastery?” Alison chatted.

“Well of course I don’t have any family of my own – the vow of chastity you know. But I still keep in touch with the uncle who gave me this watch – at least until today. I came up to London today to attend his funeral.”

“I suppose he was your favourite relation.”

“Oh no, one of my favourite relations is the relation of being less than. If it wasn’t for less-than there’d be no order in the world. It’s especially useful because it’s transitive. If I am less than you and you’re less than Ivy here then I would be less than Ivy.”

“I didn’t mean *that* sort of relation,” replied Alison, “but I do suppose less-than *is* a rather important relation. Is it your favourite?”

“Oh no, my favourite is same-size.”

“What’s that?” Alison asked. It sounded like mathematics and although she wasn’t as good at it as Emily, she was nevertheless curious. Mathematics seemed to her to be like a great storehouse of dark secrets and even if she only understood little bits and pieces she was content.

“You must be a real wizard if you’re good at mathematics,” she said in great awe, “so what is the same-size relation?”

“Well,” he said looking around the carriage. “Have you noticed that there is the same number of people in this carriage as there are seats?”

“Just a moment and I’ll count. One, two, three, four, ...”

“No, same-size is much simpler than counting. You just have to look round for a second to see that every seat is occupied but nobody has to stand up. So every seat is matched up exactly with the person sitting on it.”

“But what if somebody was sitting on someone else’s lap?”

“Then you no longer have a one-to-one correspondence.”

“I have a one-to-one correspondence with me sister,” interjected Ivy hoping that this would help to explain things. “Every second week I write to her and every other week she writes back.”

They chatted merrily for some time about numbers and whether two plus two was really four. After stopping at several stations – this time they were all the right way up – the train stopped at a station called Upney, and who do you think got on the train and into the very carriage where they were sitting. Why, it was Alison’s sister Emily!

“What on earth are you doing here, Emily?” Alison asked in amazement. How did you get into this other world? Was it through that fake red door?”

“Oh no,” there are many doors into other worlds.” said Emily. “After waiting for you for half an hour I tried the red door but, as you said, it was just a dummy. So I walked down to the end of the street where Mr Barrie used to live – you know, just across the road from Kensington Gardens – and I met this beaver, and he said”

“Don’t be silly Emily, you’re fantasising. For a start beavers only live in Canada, and besides they don’t talk. Animals only talk in books.”

“Well *this* one talked. He told me that he heard that you’d taken the District Line to Upminster. He said that there was an express tree in the garden and I could take the branch line to Upminster which Peter often used when he couldn’t be bothered flying. It was an ever so interesting tree. I asked if he would show me the way but he said he was very, very busy just now and he couldn’t. ‘But just climb up onto that first branch,’ he said and it will take you to the Minster at Upminster.”

“But we’re here in Upney, not Upminster.”

“Well, you see, I got confused. When I saw the sign on the station at Upney I thought it was the station you’d gone to. When I realised my mistake the train had left and so I had to wait for the next train – so here I am!”

“Yes, but I can’t understand how you were able to get here before we did ... unless it was all that time we spent going around the Circle Line.”

“I think you mean the Möbius Line,” interjected Brother Charles. “The Circle Line is topologically quite different.”

“Whatever,” said Alison. “So aren’t you going to sit down?”

“But all the seats are full,” said Emily.

“If we ask everyone to move up one seat,” said Brother Charles, “there’ll be an empty seat for you.”

“But won’t that mean that the last person will have to stand up?”

“Oh, how silly of me. I’d forgotten that there are only *finitely* many seats in the carriage. If there were infinitely many then there’d be no last person to miss out on a seat.”

“You could sit on my lap,” said Alison to her sister. Though, of course you’d upset the one-to-one correspondence.”

“Oh, you’re really horrid. I haven’t upset anything. Aren’t you pleased to see me?”

Of course Emily was pleased to see her. After all, she was her little sister. If only they didn’t argue so much when they were together she would have been even more pleased.

The train rumbled on at great speed, now going over a bridge, now going through a tunnel. Every so often it roared through a station, crowded with people who would have to wait for the slow train.

By now it was getting dark and the rain was coming down in parallel lines, lit by the light from the windows. The wind began to ... I was going to say ‘howl’ but if anybody has ever heard the sled dogs howling at night in the Greenland village of Kulusuk they will know that it sounds nothing like the wind. No, there is no sensible word to describe such wild weather except ...

“Oh, Alison, isn’t it positively wuthering out there,” said Emily. Of course, being the reincarnation of Emily Brontë she naturally used the word that Emily used in the title of her one and only novel, *Wuthering Heights*. Well, no *other* word describes the wild weather that was snarling outside the windows of the train.

“I’m glad I’m not out there in that blusterin’ storm,” said Mrs Dunn.

At that moment the train lurched. Brother Charles fell into Ivy’s lap. Ivy screamed. The carriage rolled over. The lights went out. There were screams from all parts of the carriage. The carriage rolled over and over. People tumbled over one another like the clothes in a tumble dryer.

The carriage ended upside down and Alison was lying on the ceiling. She was uninjured except for some small cuts from the broken glass of the ceiling lights.

All was now quiet except for a few low moans – most of the passengers were either dead or unconscious. Actually, as they learned later, by some miracle nobody was actually killed, but lots had to be taken to hospital, so you don’t have to be *too* upset.

Alison called out “Emily, are you all right?” A voice called out in the darkness “Oh, Alison I think I hit my head, and I feel very cold.”

“That’s not surprising,” said a voice that sounded like Brother Charles, except it was a lot more wobbly. “The side of the carriage is smashed open and the wind is howling in.”

Mrs Dunn called out “Oh luv that was the most exciting thing that has ever happened to me – much more fun than the roller coaster on Brighton Pier!”

So none of the four people whose names we know were badly hurt. It’s a funny thing that you care so much more about somebody if you know their name. If thousands of people in a far off land are killed in an earthquake you feel only a vague sympathy. But if the boy next door gets knocked over by a car and breaks his leg, you are horrified. And if anything had happened to Emily, apart from a sore head, Alison would have been devastated.

So let us feel a general sympathy for the moans that were starting to spring up around them as the other passengers came to their senses. Rescue workers were already starting to assist them. Our interest lies with the Christianson sisters and with Brother Charles and Ivy Dunn, so let us climb out of the carriage after them, dust off our clothes, feel our bruises, and follow them into the darkness.

They didn’t need medical help, and as they were anxious to get to Upminster, they decided to climb up the embankment to the road that runs beside the railway line to see if they could get a lift into a town.

The road was deserted, so they walked along it. They had taken a vote as to which direction to take and, as the result was two-all, they followed Brother Charles’ advice.

“But what if the next town is miles away in this direction and the last town behind us is very close?” Alison asked anxiously.

“Yes,” said Emily. “If we don’t find anything for five miles, perhaps we should double back and try the other direction.”

“That reminds me of a puzzle that my uncle once asked me,” said Brother Charles. He no longer looked like a white rabbit because his cassock was splattered with mud. Not that anyone noticed because there was no moon and it was still pouring with rain, although mercifully the wind had stopped.

“You’re on an infinitely long straight road. You know that there’s a town somewhere but you don’t know whether it’s in front of you or behind you. What do you do?”

Emily was good at puzzles and this one was easy. “Well if you make the wrong choice you might be walking forever. So I suppose you go backwards and forwards, each time going some way beyond the point you reached before. That way you’re covering both directions and *eventually* you’ll reach the town – if you live that long!”

“But how will you know when you’ve reached the point you got to before.” Alison had a point. “If it was a paved road and you had a piece of chalk you could make marks every time you went back, but on this muddy road we can barely see the sides and it all looks the same.”

As they trudged along they kept their spirits up by giving each other puzzles. Ivy didn’t join in – she didn’t seem to need anything special to keep her spirits up. By the look on her face she was obviously enjoying the adventure.

Of course, you’ll have to take my word for what her face was like, because it was so dark that you couldn’t see your hand in front of your own face. Authors are allowed to know things that the people in the story can’t possibly know.

As far as stories are concerned, authors are like God. I wonder if we're all just characters in a book that God has written. Perhaps that's how He knows what will happen in the future.

Although, I must admit that sometimes the characters in my stories answer back to me and refuse to do what they're told and the story ends up being rather different to what I had intended. I wonder if God has the same trouble with us!

After about an hour Ivy, who was walking ahead of the others called out. "Oh bother?" This was preceded by a dull thud, like somebody walking into a wall.

It wasn't a wall but, by the side of the road, there was an upturned caravan. They felt around it and at the back there was a door that should have opened sideways but now was lying upwards. Brother Charles climbed up and crawled inside and soon came out with a light that he had found.

It was one of those lamps where you remove the glass and light a wick, not one that works on batteries. They used it to explore the caravan. Of course, being 1933 it wasn't one of those large mobile homes that need a powerful four-wheel drive vehicle to pull them. No, it was a small gypsy caravan like the one that Toad of Toad Hall rode in – that is until he discovered the motor-car.

Between the four of them they managed to set the caravan upright. They had no horse to pull it, but at least they could get inside it and keep dry. Although it had turned over into the ditch the caravan was barely damaged and it was just the thing to keep the rain out.

Mrs Dunn rummaged through the drawers and to her amazement found them all full, though the china and glassware was all smashed. Whoever had abandoned the caravan had never bothered to take anything away with them.

There were clothes of various sizes which, with a bit of imagination, they managed to get into. And in the kitchen area they found packets of biscuits.

There was only one bed, which Alison and Emily occupied, head to tail. Mrs Dunn slept on the floor and Brother Charles climbed right underneath the caravan. Although the ground was quite wet and muddy he managed to find some straw in a space just under the roof that he could use as a bed. This was obviously meant for the horse to eat. If only they had one! Tiredness soon overtook the aches and the bruises and they slept deeply.

By the next morning the clouds had been pulled aside like curtains and the sun came onto the stage that we call the sky. It was a perfect day. Emily went outside and, to her amazement, she found that in a field not fifty yards from the caravan, was a horse.

Now you and I would have felt the need to go looking for the owner of the horse to ask if we could borrow it, before hitching it to the caravan. Even the caravan, although it had been left abandoned, still had an owner.

But in stories they seem to do things differently. In stories it seems to be that if you need something and there is nobody around to ask, you just take it. Didn't the hungry Goldilocks go into the house of the three bears and eat up baby bear's porridge?

Besides, if they hitched the horse to the caravan they could travel much faster than walking and, with luck, they might find the owner of the horse. The chances are that he would also be the owner of the caravan and he might thank them for having brought it back to him.

This was how they justified what they did but I dare say that if Mr Toad had been the owner of the caravan I'm not sure he would have thanked them for bringing it back. Having seen the beauty of the motor car he thought the caravan ugly and hideous and he would probably have intended for it to stay where it was until, after many years, it fell apart.

Brother Charles had had some experience with horses at St Huberts and he knew how to attach the horse to the cart and how to drive it. So off they went in the morning sun in borrowed clothes, with their own clothes flapping in the gentle breeze and warm sunshine, getting dry. The mud on the road had dried out so that fortunately they didn't get bogged.



“This is the very thing,” said Ivy with much glee. “Here today, somewhere else tomorrow!”

“I’ve heard those words before,” said Emily. “Isn’t that what Toad of Toad Hall said when he set off on his caravan journey? I remember seeing the film.”

“I don’t remember no such fillum,” said Ivy. “Is it a talkie?”

“Of course,” replied Emily, “it’s not the olden days.”

“What year d’ya think it is dearie?” asked Ivy.

“Why it’s 2006,” said Emily. “I was born in 1992 and I had my fourteenth birthday a few weeks ago”.

“What year is it Brother Charles?” Ivy called out to him at the front of the caravan where he was holding the reins.

“Why it’s 1933.”

“Yes, I thought it was,” said Ivy looking at Emily as if she was mad. “Don’t you even know what year it is?”

“I don’t understand,” said Emily.

“My dear sister, we’ve stumbled into another world, except that it’s our own world seventy years ago. I don’t understand it any more than you do.”

“Where do you think you’re going you stupid horse?” called Brother Charles, struggling to display Christian charity to the obstinate animal that had pulled the caravan off the road and onto a small track. Perhaps the horse knew that at the end of that track would be some tasty oats. He had obviously been down this track before, and no amount of pulling on the reins would persuade him otherwise.

“Oh well,” said Brother Charles with resignation. “Perhaps he’s taking us to his owner.”

“Maybe we’ll get a nice breakfast there,” said Ivy Dunn. “My stomach’s as ‘ollow as a cave.”

Presently they saw, working in a field near the road, a farmer, ploughing the fields. They tried to stop so that they

could ask directions but the horse was determined to keep going. So Brother Charles had to jump out and stand in front of the animal.

“Hullo there! Would this be your horse?”

“Nayyy,” he said sounding a bit like a horse himself.

“This be my ‘oss ‘ere at t’other end of the plough.”

“Well we found this horse and we’re trying to find its owner.”

“You could keep going ... or you could turn back,” said the farmer helpfully. “It’s all the same to me.”

They then noticed that the farmer had a set square and a pair of compasses in his hand and a bundle of papers. They wondered what he was doing.

“I be ploughing parapel furrers and I ‘ave to make sure that the furrers don’t meet. Did you know that for every line and every point not on that line there’s eggsactly one line through that point parapel to that line?”

“My word,” said Brother Charles. “It’s old George, the Farmer Geometer. I’ve heard of him. That was Euclid’s Parallel Postulate he just enunciated.”

But Farmer Geometer went back to his work and so they drove on for another ten minutes. The road got narrower and steeper and bumpier and soon they had to get out and push the caravan from behind. After another five minutes they passed through some ornate gates which bore the name ‘Paradox Castle’.

PARADOX CASTLE

Our four adventurers left the caravan outside and walked up the stone steps to the massive front door. In front of the door was a large mat with the word ‘WELCOME’ woven into it.

There was a bell pull, and a sign ‘PULL ME’ next to it, so Brother Charles gave the bell a hard pull, and immediately there was a loud bang as a small cannon ball shot across their heads.

“Not a friendly castle,” said Ivy. “Why did the mat say ‘welcum’?”



“It reminds me of the welcome James Balfour received at the House of Shaws,” said Alison. “I wonder if Uncle Ebenezer is at home.”

Brother Charles had another idea. “Perhaps the castle is deserted.”

Alison thought otherwise. “How can you say that? As soon as you rang the bell someone fired at you.”

“It’s probably a booby trap. The shot came the moment I rang. Let’s try the door.” He pushed it and it swung open.

Now to you it might seem strange that the front door would be unlocked. But remember that the three bears left

their house unlocked. The only doors that seem to be locked in these strange worlds are secret inner doors with the words ‘OPEN ME’ painted on them.

They walked in cautiously, half expecting another booby trap, but nothing happened. They found themselves in a large square entrance hall. At the end there was a large red door, with the words ‘OPEN ME’ printed in large letters.

“It’s probably locked,” said Emily. “We’ll have to go looking for a key.”

“No, it’s worse than that,” said Alison. “It’s not a real door. It’s only a painted door – a façade. Only the key-hole is real.”

“What’s the use of a real key-hole in an imaginary door?” asked Emily.

While this was taking place Ivy had investigated the only piece of furniture in the room – a table in the centre. On it were five objects, each with a label tied to it by string.

There was a strange looking brown glass bottle. Its label said ‘DRINK ME’. There was a large key whose label read ‘WIND ME’. A blunt pencil had a label that said ‘SHARPEN ME’ and a pipe, the type you use for smoking, had the label ‘DRAW ME’. Finally there was a blank sheet of paper attached to a blank label. They tried the large key in the imaginary door. It fitted the real key-hole but it refused to turn.

“The obvious thing to do is to drink whatever is in the bottle,” said Ivy reaching for it.

Brother Charles had only just noticed what Ivy was about to do. “Don’t drink out of that bottle,” he screamed as he pushed her away.

“That was a close thing,” he said. “You probably don’t know that that is a Klein bottle. It’s like a three-dimensional version of a Möbius Band. It’s a one sided surface and so the outside and the inside are all one. If you had drunk it then the whole universe would have gone inside you!”

“But that would have meant that she would have been inside herself,” said Alison. “How would you explain that?”

“I wouldn’t,” said Brother Charles. “I’d much rather try to work out what to do with the other four items.”

“Well, we have a key that says ‘WIND ME’. That’s quite a paradox. A key can’t wind itself,” said Emily.

Alison was equally puzzled. “And a blunt pencil that says ‘SHARPEN ME’ is no good without a pencil-sharpener. And we can’t draw the pipe until we somehow manage to sharpen that pencil”.

Ivy was examining the key and she noticed that it ended in a hollow cylinder. “P’raps this ‘ere pencil goes inside the key.” She tried it and discovered that the key was also a pencil-sharpener. So she sharpened the pencil.

“Now, who’s going to draw the pipe?” asked Brother Charles.

Emily drew a pretty good picture of the pipe. “Now what?” she asked. It was then that she noticed that the table now had a drawer. “I’m sure there was no drawer before.”

“Maybe it needed another drawer in order to appear. That was you Emily.” Alison opened the drawer, and inside was a tiny little key. It was clearly too small to fit the keyhole in the painted door. The problem was to find the small keyhole.

They examined the room, and the table. They even tried to find a tiny keyhole in the pipe. Eventually they found one in the large key.

They wound it up and the large key started to turn. At least the end part with the teeth started turning but the handle did not.

"I get it," said Emily. "You put the key in the lock and wind it up and it turns the key by clockwork. If only the door was real."

Just then Brother Charles had a bright idea. He held up the picture of the pipe. "What's this?" he asked.

"Why any dummy can see it's a pipe," said Ivy.

"Well no, it *isn't* a pipe – it's a *picture* of a pipe. So it's an imaginary pipe. Now you've heard that two negatives make a positive."

"Yes," said Ivy. "At least that's what they done taught me at school, but I didn't never really know what it signified."

"There you go, Mrs Dunn. If you didn't never know then you always *did* know. Two negatives make a positive." It was Emily who pointed out that Mrs Dunn had actually illustrated the point.

"I knows what I mean and if I says something, I mean what I mean it to mean."

"Oh, don't be so mean to Mrs Dunn," said Alison to Emily. "Go on Brother Charles."

"Well just as two negatives make a positive when you multiply them, so two imaginaries become real. Take the imaginary pipe and hold it against the imaginary door."

They did this, and to their amazement the imaginary door became a real door! They took the key, wound it up and inserted it into the keyhole. With a whirring of clockwork the large key unlocked the door.

Emily pushed the door open and looked in. The others followed her in. It was a vast hall filled with strange creatures seated on chairs. They looked a bit like people but they had strange flat faces, and each one had an arrow on their lap.

It might have been a church, because it had stained glass windows. It might have been a school because the walls were covered with blackboards containing what looked like algebra. But it was probably a court of law because it was presided over by what looked like a judge. Emily, Alison, Brother Charles and Mrs Dunn stood at the back of the church/classroom/court and watched with interest.

“Silence in court.” The judge banged a pair of compasses on the bench and used them to draw a circle. Standing in front of him was the most beautiful girl that Alison had ever seen. She was dressed all in white and was carrying a bunch of snapdragons. Next to her was what appeared to be her intended husband.

“We are gathered here to unite these two in a logical union. Marriage has been ordained for the mutual comfort of the two and for the production of theorems. Do you, IF, take THEN to be your lawful conclusion?”

She nodded, “I do.”

“Do you, THEN, take IF to be your lawful assumption?”

“I do.”

So, it must have been a church after all.

“I therefore sentence you both, for the terms of your natural lives, and for all eternity, to become a theorem in the world of mathematics.”

Everybody stood up and cheered and waved their arrows. Someone standing near the back asked Alison if she was an IF or a THEN.

“You look like an IF but appearances can be deceiving.”

“I’m neither. I’m a girl”

“You must mean that you’re a CONJECTURE, an arrogant IF that’s pretends to be a THEN when it’s really only an IF. He raised his implication arrow and might have stabbed her if she hadn’t been too quick for him.

Alison ran down the aisle and the other three followed her. They ran out of a side door that would have been a vestry if this had been a church. They locked the door behind them and trembled at the clamour of voices outside. “Disprove the CONJECTURE, they called. Is there a COUNTER-EXAMPLE in the house?”

Fortunately there was another door to this ante-room. They ran out and found themselves at the bottom of a spiral staircase. They ran up the steps as fast as they could. It seemed to go on and on, ever upwards, round and round.

“Perhaps it’s an infinite staircase,” said Emily. It might never end.

They noticed that carved on the edge of each step was some writing. On each step they read:

AT LEAST ONE OF THE HIGHER STATEMENTS IS
FALSE

“Well it can’t be an infinite staircase after all,” said Brother Charles as he examined the steps.

“How can you work that out,” Alison asked.

Brother Charles stopped and pointed to one of the inscriptions. “If this statement is false, then all the following statements must be true.”

“Why?” asked Alison.

“Well, if it isn’t the case that there’s a false statement after this one then they must all be true beyond this false one.”

“I think I see that,” said Alison.

“Well how can that be? If all the remaining ones are true then none of them are!”

“That sounds like nonsense,” said Alison.

“And so it is. If this step is false we get nonsense.”

“So it must be true. But then you could use the same argument at each of the higher steps, so they are all true.”

“In which case none of them are.”

“So we have a paradox.”

“Only if there are infinitely many steps. So there must be a last one.”

“I don’t understand a word youse all are saying,” said Ivy. “What I does knows is that we seem to be agoin’ round in circles. I’m sure I’ve seen that water stain on that stone wall. We’ve done passed it afore.”

“But how can that be,” said Alison. “We’ve been going upstairs the whole time. Maybe there just happen to be lots of very similar looking stains. What we should do is to keep

someone here while the rest of us go further up the stairs, and when we reach that stain we'll know whether it really *is* the same one."

"Good idea," said Emily. "I'll stay."

So the rest of them continued going up and after a couple of minutes they met up with Emily again.

"That proves that Ivy is right. We're somehow going around in circles."

"Not necessarily," said Alison. "It could be that Emily found a short cut to the next level. She's tricky like that."

Either way, finite or infinite, there was no point in going up. They had to go back down and face the music.

The strange thing was that going down they came to the ante-room in no time at all. Going down there were far fewer steps than going up! But what was more important was the fact that the door to the vast hall was unlocked and the hall was completely empty.

They walked to the altar, or was it the judge's bench? They noticed there was a book with the label 'READ ME' on the front. Inside was the title of the book: *Proof That God Exists*.

It was a very thin book. In fact, apart from the title page, there were only two other pages.

On page 1 it said "God exists". On page 2 it read "Everything in this book is false."

"That's a pretty silly proof," said Emily.

Brother Charles was not so dismissive. "I've seen this book before in my seminary. Think about it logically. Page 2 is either true or it's false."

“Even I can unnerstand that,” said Ivy.

“Could page 2 be true?”

“No, because that would mean that it is also false,” said Alison. “It says that *everything* in the book is false and, if that were the case it *itself* would be false.”

“So page 2 must be false.”

“I suppose so, said Emily.

“So it is false that everything in the book is false.”

“Oh dear, that sounds like your double negative again.”

“If it’s false that everything is false then something is true.”

“Oh I get it,” said Emily. “One of the two pages is true, and it’s not page 2 because we said that can’t be true. So it must be page 1 that’s true. So God exists.”

“Wow,” said Alison. “I never thought you could prove that God exists by simple mathematics. But wait a minute. If I changed page 1 to ‘Santa Claus exists’ I could prove *that* true as well. Something funny’s going on here.”

“Indeed it is,” said Brother Charles. All it really proves is that logic has its limitations. There’s an old story that was old even in the days of St Paul. In his letter to Titus he wrote about a Cretan, called Epimenides, who said “all Cretans are liars.”

“I get it,” said Alison excitedly. “In particular he was saying that he, himself was a liar. So if that was true then it was false. But if it was false then it was true.”

“But, sister dear, to be a liar doesn’t mean that you *always* lie. Perhaps that particular statement was true. I mean I’m sure you would agree that everybody in the world has told a lie at some stage. If ‘liar’ means having *sometimes* told

a lie then I can say truthfully that all of us are liars. That statement just happens not to be a lie.”

“But Jesus wasn’t a liar,” argued Alison.

“What we call a lie means more than the fact that is untrue – it means that it is both untrue and we know it to be untrue. Jesus on the cross said, ‘my God why hast thou forsaken me’, implying that God had indeed deserted him. But I believe that God never left him. So what He implied was false, but you could hardly call it a lie!”

“Would you two stop talking nonsense,” said Ivy, “and let’s go look for the kitchen in this ‘ere castle. I’m beginning’ to get hungry.”

There was another door that led out of the court, or maybe it was a church. Anyway it led down some stairs to the kitchens. The only person there was the cook, a rather short round man with big eyes and a shiny round bald head. He reminded Alison of Humpty Dumpty.

Ivy went up to him and said, “sir, would you be so good as to cook me an ’am and cheese homelette.”

A look of fear came into his eyes and he stepped back two paces.

“Don’t tell me you’re really Humpty Dumpty?” asked Emily. This seemed to be the only explanation for his bizarre behaviour.

He pulled himself up to his full height. “Do I look like an egg?” he asked with great affront.

“Not at all,” said Emily reassuringly, though with still some doubt. “It’s just your reaction when Ivy mentioned the word ‘omelette’.”

“Oh, that. It’s just that I detest eggs. I never cook eggs, I never eat eggs. I can’t bear to be in the same room as an egg. Even the word ‘omelette’ hits me like an arrow.”

“Is it somethin’ you et as a child?” asked Ivy.

“Oh no, I used to love eggs, that is until I was old enough to read *Through The Looking Glass*. I thought you had to read it backwards by holding it up to a mirror, so it took me a while before I got to the bit about Humpty Dumpty. I was silly like that. I got the book *Now We Are Six* on my fifth birthday and I felt very guilty reading it because I wasn’t yet six.

Anyway I learnt that Humpty Dumpty was an egg. And since my parents had thought it a great joke to christen me Humphrey I was devastated.”

“But lots of people are called Humphrey and nobody associates them with Humpty Dumpty.”

“If your family name was ‘Dumphrey’, would you call your baby son ‘Humphrey’? You can imagine what I had to put up with when I went to school – Humphrey Dumphrey. I know every egg joke ever created by a cruel schoolboy.”

Alison felt great sympathy for him. At least her name wasn’t Wonderland, and even so she got to hear more Alice jokes than she wanted. If only Lewis Carroll had realised what pain he would cause for future generations of children he might have concentrated on his syllogisms.

“How did you cope?” Alison asked him. “Did you fight them to prove that you weren’t so fragile?”

“Oh no, I played along with their jokes. I even made up a little song about it. Would you like to hear it?”

“Yes please!” said Alison. So he stood on the table and sang this song.

There existed an egg who sat on a wall,
And the wall being short implies this story is tall.
Now if that fat egg had had a great fall
Or slipped off the top, but not jumped, then not all
The king's horses and all the king's men,
If they worked through the day and the evening, then
They could not succeed if and only if when
They attempted to put Humpty together again.

“I think I prefer the original words,” said Alison. “No offense. It’s just that I find all the ‘ifs’ and ‘thens’ very confusing.”

“Oh I got all that from one of Charles Dodgson’s books on logic.”

“Who’s he?”

“Why that was Lewis Carroll’s real name. He was a mathematics lecturer. Would you like me to sing you another song? It’s called *The Asterisk Song*.”

He didn’t wait for a reply, but burst into song. The tune was very familiar.

Sprinkle, sprinkle asterisks,
Made of intersecting sticks,
How I wonder why your friend
Gives the reference at the end.
Up above the text so high

Like an astrum* in the sky.

* astrum means 'a little star'.

I've written out the song as best I can. Where I put an asterisk Humphrey had let out a sort of high-pitched squeak.

"I don't think I understand that," said Emily. "Wasn't Asterisk a sort of barbarian who lived at the time of the Romans?"

"No dear," said Alison, pleased that she could put her little sister right. "An asterisk is a symbol." Alison knew all the symbols on her father's computer.

"That's right," said Humphrey. "It's that little star you put next to a word that you want to explain at the bottom of the page. It's a very polite little symbol. It doesn't interrupt the flow of the text but waits till the bottom of the page to give its explanation."

"But why did you say 'up above the text *so high*'?"

"That's poetic licence, though you must admit that the asterisk is a little higher than the text."

"But what did you mean when you said it's a 'very polite creature'? You make it sound like a person."

"Oh no, symbols aren't people as such, but they live in the village of Glyph, which is not far from here? I could take you to them?"

"Do we have to walk?" asked Mrs Dunn. "I'm right tired out, as well as fair starvin'."

"Oh, not at all," said Humphrey. "Not when all the King's men and all the King's horses are at our disposal."

"At this he took a large skillet and a large spoon and banged one against the other as loudly as he could. He then

motioned them to follow him out of the door to the outside. There they saw gravel drive along which came a regiment of horsemen. At the rear came the royal coach.

“The King won’t be requiring this today. He’s in bed with a cold,” said Humphrey. “Allow me to help you in.” He stepped towards the coach but unfortunately the royal prince, a bonny boy of four, had been playing in the drive that morning. Carelessly he’d left a ball on the ground. He was a most untidy little boy, and I don’t know how many times Nanny had told him to put away his toys when he’d finished playing.

So Humphrey Dumphrey stepped on the ball and fell over. He was so upset he went to pieces. I don’t mean that he literally fell apart into little bits, like an egg might have done. When we say that somebody ‘goes to pieces’ we simply mean that they lose control of themselves and behave in a very un-adult way.

Humphrey let out a loud wail and burst into tears. Some of the King’s men came to help him up, but he was inconsolable. He climbed into the coach, still screaming. Alison climbed in after him to try to calm him down, and the others followed. The coachman cracked his whip and the whole procession moved off.

INFINLAND

They never got to the village of Glyph. After half an hour of travelling along a dusty road they came to a castle. It looked very much like the one that they had just left.

“We seems to be agoin’ roun’ in loops agen!” exclaimed Ivy Dunn.

“Yes,” said Alison. “It’s just like on the Möbius Line. Except shouldn’t it be upside down?”

But Brother Charles always knew how things work in this strange world. “No, it’s a different castle that just happens to look the same.”

Emily agreed. “Perhaps we’re just on a different level. Alison, don’t you remember when we were in that multi-story car park going round and round looking for an empty spot. Mother thought it was silly going round and round because we could very well see that all the places were full. She hadn’t noticed that by then we were on level five. And eventually we *did* find a parking spot on seven.”

“But this is open country. There’s nothing multi-story about it,” Alison said in return. “We can check it out if we stop and scratch a mark in the dirt.”

Brother Charles told the driver to stop and he got out. Finding a small branch he drew a cross in the dirt. Then they went on.

After another half hour they came to another Paradox Castle, or was it the same one? They stopped and looked where the cross had been drawn on the road. There was a mark, but this time it was a nought.

“See,” said Emily, “it’s a different castle. Let’s go on. Another half hour brought them to a castle that looked exactly like the ones they had seen before. And on the road, near the gate, was a cross.

“Some un’s playing ‘oughts and crosses with us,” said Ivy.

As they continued on Alison thought about their predicament. “Perhaps there are two identical Paradox Castles, one with a nought drawn on the road outside and the other with a cross. And we are *indeed* going round in circles.”

“Let me explain why we’re confused by telling you a little story,” said Brother Charles. Once upon a time there was a tiny insect that had somehow landed on a sheet of postage stamps.”

“But stamps don’t come in sheets,” said Alison. “They come in rolls of 100 in a cardboard box.”

“Don’t forget this is 1933,” her sister replied. “They didn’t have rolls of self-adhesive stamps back then.”

“The only stamps I know,” said Brother Charles, “come in sheets and you tear them off and lick the back and stick them on.”

“Oh, how unhygienic.”

“Don’t be silly Alison, people licked stamps for over a hundred years and it didn’t kill them.”

“Well this was a large sheet of stamps and each one had the queen’s head.”

“Do you mean Queen Elizabeth?”

“Good gracious no. Stamps weren’t invented *that* long ago!”

Emily realised that he thought she meant Elizabeth the First. It was 1933, after all.”

“The head was, of course, of Queen Victoria, who died about thirty years ago. So, all the stamps were identical and, as the little insect moved around, she thought she was going round in circles, whereas it was a different stamp each time.”

Emily seemed to be working something out. “And the fact that she seemed to be going round in circles left and right, and seemed to be going round in circles up and down she would believe that she was crawling on a torus!”

“Aint that a sort o’ bull,” said Ivy. “I thought you was sayin’ the little mite was on a sheet o’ stamps.”

“No, Taurus is a constellation of stars,” said Alison.

“Silly sister,” said Emily. Don’t you know that ‘torus’ is the mathematical name for a doughnut shape in topology? Well if the insect was really crawling around the largest part of a doughnut it would be going around in circles. And if it went at right angles it would go round in circles in a different direction.”

“Well I can’t imagine that a tiny insect would know about topology,” said Alison. “But if it had any intelligence it would eat all the sugar on the doughnut and see if, when it came back to the same spot, where the sugar had been eaten.”

“But sister dear, have you forgotten that she wasn’t on a doughnut but on a sheet of postage stamps. There’s not usually much for an insect to eat on a sheet of postage stamps.”

“It’s only a story to illustrate my point,” said Brother Charles bringing the discussion to a close. “The point is that we may or may not be back at the original Paradox Castle.

Why don't we stop and go inside. That might help us find out."

"I'll just stop here in the coach," said Humphrey. "My ankle has started swelling up. I'll just amuse myself by making up songs. Sing a song of half a crown ..."

As the others walked away, they could hear his voice getting fainter and fainter, until they could no longer make out the words. But in case you are interested in his song, here are the words. You can see how the reader of a book has many advantages over the characters in it. They can know things that the people in the story don't.

Sing a song of half a crown,
A handbag full of rice –
Two and forty raisins
Baked in a slice.
When the slice was cutten
The raisins did come free.
Wasn't that a silly dish ...

Now what could possibly rhyme with 'come free'? Oh, I have it.

To set before Dumphrey."

By now the others had walked up the stone steps to the massive front door. In front of the door was a large mat with the word 'WELCOME' woven into it. There was a bell pull, next to a sign saying 'PULL ME', so Brother Charles gave it a hard pull.

Now you may be thinking that it must be the original Paradox Castle in the last chapter. In fact it could be that this is one of those books where the text repeats over and over, giving the impression of going round in circles. But when Brother Charles pulled the bell nothing happened. Does this prove that it is a different castle?

It's interesting to ask oneself what 'different' means. We're told that the body replaces each cell every ten years and so, in a sense, you are a different person to who you were ten years ago. (This is especially true if you are only nine!)

Or imagine that your mother was an identical twin, but you had no idea that she was. So it could be that the mother you wake up to is not your real mother but her twin sister who had been smuggled into the house while you were asleep, while your real mother was sent away. This could account for the fact that some days she's really nice and other days she's quite cranky. Sometimes people say "Oh, I'm not myself today."

Is the fourth word in this sentence the same word as the last word? In one sense it is. Both are the word 'word'. Yet they're not quite the same really because they are in different places in the sentence. You see how difficult it can be to know whether one thing is the same as another.

Well this castle could be the same one that they had been in before. But if we find different people inside, or nobody at all, that doesn't prove that it really *is* different. In this case there *was* somebody home, and she was quite different to anyone they had ever seen before.

The door opened and a small girl answered. She was quite pretty, but very, very small, like the diminutive Polly in Charlotte Brontë's novel *Villette*. If you want to read a wonderful description of a pretty, but tiny, little girl I suggest you read the first chapter of that book.

I can't quite tell you how old she was. It's hard when someone is so small. By height she was only about four years old. But by shape she was probably closer to sixteen. You know how girls all change shape as well as get bigger when they reach their teens. This girl had the shape of a sixteen year old.

"Hello," said Alison. "We were wondering if we could come inside to see if this is the same castle as the other one." There, you see, once again we find Alison talking to a stranger. But I suppose her mother wouldn't have minded because this girl looked so sweet and so pretty.

"Well, if the other one really is the *other* one," she said, "then it must be different, by the definition of the word 'other'. But do come in. We can't be standing out here all day discussing the meaning of words."

As they came into the hall they noticed that she was holding out her right hand as if to greet them. Alison went to shake her hand, but the girl shrieked and jumped back.

"Oh don't do that, you might crush it."

"I'm sorry I didn't realise you were holding something." Indeed it *did* appear that her hand was quite empty.

"What is it that you thought I might squash?"

“You probably can’t see it because it’s so very, very small.” They all peered closely at her outstretched hand but could see nothing.”

“It’s the land I come from. You see, I’m an Infin and this is Infinland I’m holding.”

Now I know what you are going to say. In fact both Alison and Emily both said it at the same moment. How could a country be so much smaller than one of its inhabitants. The girl offered an explanation.

“You’re probably wondering how a country can be so much smaller than one of its inhabitants. Well the other day I came across a bottle with the label ‘DRINK ME’ and I was foolish enough to drink it. And I grew and grew until I became so large that I popped out of my world into this one.”

Alison thought it strange that such a tiny person could describe herself as ‘so large’ but it was true that she was much bigger than her country, if indeed there was anything at all in her hand.

“If only I knew how to get back!”

“Can’t you find another bottle marked ‘DRINK ME’. It might have the opposite effect.”

“I’ve looked everywhere in this castle and all I could find is this jar marked ‘VANISHING CREAM – RUB ME IN’. But I was too scared to try it. I want to become smaller so that I can get back inside my country but I don’t want to disappear altogether.”

Emily took the jar from her and opened it. “I don’t think you’re likely to do that. Matter cannot be created or destroyed. It can change into energy and back again. But the law of conservation of energy says that in one form or

another it will always exist. The worst that can happen is that you would be changed into a beam of light. But with a bit of luck you might become so small that you *appear* to have vanished. Isn't that what you want?"

But before the tiny girl had a chance to answer Emily had started rubbing the vanishing cream on her arms and legs – and it worked perfectly. She disappeared!

"Oh Emily, what have you done?"

A very small voice came from the floor. "I'm still here, but I'm just too small to be seen by the naked eye. Does anyone have a microscope handy?"

But nobody did. "Well perhaps you can find a sheet of postage stamps. I could crawl about it and test the theory about it looking like a torus."

"Don't worry about that, just look for a bottle marked 'DRINK ME'."

"No, I can't see anything. It's just empty space."

"Well I guess we'll have to come down after you," said Alison as she rubbed some vanishing cream onto her skin.

One after the other they all became microbes, or whatever you call miniature creatures that need a microscope to be seen. The last one to do this was the tiny girl, and as soon as she did they suddenly found themselves on a street.

You see, what happened was that once the tiny girl (I wish I knew her name because I'm tired of having to refer to her as 'the tiny girl') became infinitesimal in size she dropped the speck of dust she was carrying and this rolled on top of the others until they found themselves inside the speck of dust. This was Infinland. They picked themselves up, Brother Charles, Ivy Dunn, Emily, Alison and the tiny girl.

“It’s high time we were introduced,” she said, “I’m Tess – Tess Schröder.”

“I’m Emily with two dots over the E,” said Emily. “And this is my sister Alison. She’s older than I am but she doesn’t know quite as much. You see I read an awful lot. And this is Brother Charles and Mrs Dunn,”

“Call me ‘Ivy’,” said the latter.

“I like the name ‘Tess’,” said Alison. It’s a good strong name that can’t be shortened.”

“But in fact it’s short for my full name. I was christened ‘Infinitesimal’. But I’d prefer it if you stick to ‘Tess’.”

“So this is where you live,” said Brother Charles looking around. It was a town, with pretty houses on each side of the road.

“My house is a bit of a walk from here. Come, I’ll take you to it. Mum will be so pleased to see me.”

It seemed to be a very long street. They couldn’t see the end of it. Mrs Dunn noticed the number on one of the houses. It said 729143. Next door was 729145.

“Lordy, what a long street,” said Ivy. “How many ‘ouses are there in this street?”

“Infinitely many,” said Tess. “That’s why we’re called Infins. You see it’s very useful. When someone grows up and gets married they move out of home and set up on their



own. They move into the house next door. Those people move to the one next to them and so on. In the end everyone gets to have a house to live in.”

“But what about the family at the end of the street?” Alison was worried about the fairness of the arrangement.

“Oh, nobody lives at the end of the street. The streets are infinite so there *is* no end.”

Alison was confused, but Emily got the idea. “It’s like saying ‘what about the largest number?’ There isn’t one.”

“But you can’t create an extra house just by moving people around. If you had the same number of houses as families and you suddenly got an extra family you’d need one more house. That would mean *building* an extra one.”

“In your world perhaps, but not when there infinitely many houses.”

“But the arithmetic would be all wrong. If you had N houses, and you needed one more, that would be $N + 1$. If you didn’t need to build a new house that would mean that $N + 1$ is the same as N .”

“And what’s wrong with that?” asked Brother Charles. “With finite numbers, like the ones we’re used to, $N + 1$ is bigger than N . With infinite numbers they’re the same.”

“But if you start with $N + 1 = N$ and take N from both sides you’d get $1 = 0$.” You see, although Emily knew a lot more mathematics than Alison, Alison *did* know the basics of algebra.

“All that shows is that you can’t do subtraction with infinite numbers.”

“My head is spinning Brother Charles. Now you’re talking about infinite numbers as if there’s more than one

infinity. I know about infinity and you write it like this.” And she drew an eight on its side, which is the mathematical symbol for infinity. It looks like this:



“How can there be more than one infinity?”

Brother Charles knew a thing or two about infinity. He knew, for example, that there are different sizes of infinity – bigger and bigger infinities. (No, I’m not making this up.) Having just one symbol for ‘infinity’ is rather silly, like counting “one, two, three, many, ...”.

But Alison was a bit sceptical. “How can I know if you’re correct about there being infinitely many houses in this street? After all if I keep walking I’ll never be sure whether there really *are* infinitely many houses. There might just be a very large number of them and I simply haven’t reached the end.” Emily wished *she’d* thought of this.

“You just have to take it as an axiom,” said Brother Charles. “That means you have to believe it, just as I believe there’s a God.”

“That’s not good enough,” retorted Emily. “An axiom is a self evident truth. Like the axiom in geometry that says that given any two different points there’s one and only one line between them.”

“Can’t you have one line curving upwards and another curving downwards?” asked Alison.

“I’m talking *straight* lines here.” That’s what Euclid meant by ‘line’. “If I plot two points there’s clearly one and only one *straight* line I can draw from one to the other.”

By the way, ‘Euclid’ is pronounced ‘yooklid’. This is important in case you ever meet him. Of course that’s

unlikely because he lived a couple of thousand years ago. He's known for doing geometry in a very logical way, starting with a list of axioms. These are statements about points and lines that you have to accept without proof. Then he built up all sorts of geometric facts based on these axioms.

"Why is that particular axiom obvious?" Brother Charles appeared to be teasing Emily. She was a bit put out by the fact that he seemed to be suggesting that she didn't know what she was talking about. Alison smiled to herself.

"Well I tried it once and I proved that it was true.

"How many pairs of points did you try it out on?"

"One pair – you only need to do it once."

"But the axiom says *any* two distinct points. Did you test it for *every* pair of distinct points?"

At this point Emily went red in the face. It was bad enough to be shown up, but to be shown up in front of her less intelligent sister. "I wish Alison would stop smiling," she said to herself.

But Brother Charles was gracious. "I don't mean to make fun of you. There is a sense in which that axiom is obvious. Euclid himself thought so. But if you think hard about it you must admit that there is still a leap of faith involved."

"But I always thought that you can prove *everything* in mathematics. That's what's so good about it. You don't have to take things on faith like you do in religion."

"Well, I agree that in mathematics there's an awful lot of proof. We have to thank Euclid for that. But remember you can't prove something out of nothing. You have to start somewhere."

While all this was going on, Ivy wasn't listening. It all sounded like goobldey-gook to her. She was focussed on looking at the flowers in people's gardens as they walked. She loved flowers and felt that they were much more interesting than axioms and proofs. And I suppose, in a way, she's right. At the end of the day you can't smell an axiom or plant a proof.

These activities occupied them and in no time at all they came to the house where Tess lived. She opened the door and her mother appeared, and screamed with delight and gave Tess such a big hug that she nearly squeezed the breath from her.

One thing I forgot to tell you was that the vanishing process worked differently for Tess than it did for the others. She was reduced a little less than they were, so that in Infinland she was quite tall for a sixteen year old. This meant that instead of looking down to her as they had done in Paradox Castle, they now looked up to her.

Mrs Schröder made them a lovely tea with hot muffins and scones. As the others sat around the fire Mrs Dunn asked if she could go out to look at the back garden. She came running back into the parlour with a look of excitement on her face. What a lovely view you have from the back with all those trees and flowers. Why it seems to go on forever.

"It *does* go on forever," said Mrs Schröder. "This is Infinland where everything is infinite. What you see is the Great North Park."

"But you're so lucky that your street is the one that backs onto the Great North Park," said Mrs Dunn.

“Not at all. My neighbours on the other side of the road have the Great South Park behind them.”

“Well people in the other roads wouldn’t be so lucky.”

“But there *are* no other roads in Infinland – just this, the East-West Road.”

“Well if there are so many of you in Infinland wouldn’t it have made a lot more sense to go in two directions and have streets all over?”

“But then we wouldn’t have had that wonderful open space. There was a time when that was how it was arranged. That was the time of King Aleph, long before I was born.”

“You don’t mean Alf, as in Alfred?” asked Alison.

“No,” answered Mrs Schröder. “It’s Aleph. I believe that in your world Aleph is the first letter of the Hebrew alphabet.”

Just then Brother Charles interrupted. “May I be permitted to show you how he signed important documents?” He was given some paper and a pencil and he drew the following.

ℵ₀

“The squiggly bit is ‘aleph’, and because he was the first king in his line he called himself ‘Aleph the Zero’th. His successors were to be called Aleph the First, and so on.”

“I’ve seen that aleph before when I was fiddling around with the special symbols on Dad’s computer,” said Alison.

Emily, with her logical mind, objected “but shouldn’t he have been Aleph the First? I mean you don’t count from zero.”

“But of course you do,” replied Tess. “In kindergarten we always start counting finite numbers zero, one, two, three, ...”

Brother Charles had some more to add. “Aleph zero is the name of the smallest infinite number.”

Emily had vaguely heard about this from her father who, by the way, is a professor of mathematics. She wanted to show that she knew all about these aleph numbers. “And then after that you have aleph one – it’s the next biggest infinite number.”

“Anyway I was telling you about King Aleph Zero,” said Mrs Schröder. “He was not a very nice king. He was very selfish and very proud. In those days there were roads all over Infinland. Some went north-south and others went east west in a criss-cross sort of way. In the middle of it all was the king’s palace with a rather small garden. This was the only open space in the whole kingdom and King Aleph wanted more.”

“Just like a king,” said Emily.

“So he decreed that from a certain day all the Infins should only live on the one East-West Road. Those who were living on other roads had to move to the East-West Road.”

“Were there enough empty houses on the East-West Road for them all?” asked Alison.

“Oh no. Every house was occupied.”

“Well how could they do it?”

“Oh it isn’t too difficult in an infinite world. The King sent a deputy around to every house. He went round in a spiral fashion around the palace, moving further and further out. In this way he could be sure to visit every house.”

“But that would have taken him forever,” said Alison.

“It did.”

“But if there are infinitely many houses to visit that would take an infinitely long time, which would mean that he’s still going.”

“No Alison,” said her sister. “In an infinite world you can do infinitely many things in a finite time.”

“That’s right,” said Mrs Schröder. “At each house he gave out a number: 0, 1, 2, 3 and so on forever. That was to be their new address on the East-West Road.”

“But what about the people already living there?”

“They just moved further out. I expect the traffic must have been unimaginable as they all moved on the same day. But eventually all the houses not on the East-West Road became vacant and were then knocked down. King Aleph was then able to take over the land on either side and so had two Great Parks, one on the north and one on the south. None of the commoners were allowed to use this open space. What’s worse is that the only way from the East section of the East-West Road to the West, and vice versa, was through the King’s palace gardens. He built a toll booth there and collected a great deal of money. Of course people had to travel further under this new arrangement. Next door neighbours would find themselves miles apart after the Great Move.”

“Sumun should’ve kicked the King out, like they did in the Frenchy country during their Revolvathon,” said Ivy.

“Well that’s what happened,” said Mrs Schröder. “There was a Great Infin Revolution, led by Count Abel. It wasn’t easy, but he thought of a way to do it without

bloodshed. You see, it was written down in the Great History of Infinland that the King was not only King of Infinland, but he was also Lord of Committees.”

“What sort of committees?” asked Alison.

“Why all sorts. You see we Infins just love to form committees. Whenever some Infins get together they spontaneously form themselves into a committee. There is the blue-eyed committee and the committee of all people born on a Wednesday. Some committees contain only one Infin and there is even the Empty Committee.”

“What’s that?”

“That is the committee that has nobody in it.”

“What a silly idea!”

“Well it started out as the committee consisting of every Infin who could jump over the wall that King Aleph built to keep the common Infins from going into the Two Great Parks. The wall was so high that nobody could do this so it became the Empty Committee. If a few Infins are in a bus queue they instantly form themselves into a committee. Now every committee has to have a chairman.”

“Or chairwoman.”

“How did they elect a chairman of the Empty Committee?”

“Elect? Oh, no there was never any election. The King had to *appoint* the chairman of every committee. It was his royal prerogative and also his royal responsibility. If ever he was unable to appoint a chairman he lost his right to rule.”

“Well how did he get on appointing a chairman for the Empty Committee?”

“That was easy, because the chairman doesn’t actually have to be a member of the committee. There are internal chairmen and external chairmen. An internal chairman is someone from within the committee and an external chairman is someone from outside of the committee.”

“If I was the king I would have appointed myself as chairman of *every* committee. That would make the job very easy.”

“Oh, I forgot to tell you that no Infin is allowed to be a chairman of more than one committee. If ever it was discovered that the King appointed someone as chairman of more than one committee he would lose his right to rule – it’s written into our constitution.”

“That would have made his job more difficult. He would have had to keep an accurate record of all the chairmen he’d appointed so that he didn’t double up.”

“Well he claimed to have had it all worked out in advance. You see he had this huge book in which he said he had listed all possible committees and their chairmen. So all he had to do was to look up this book.”

“Did anyone ever see inside this book?”

“Oh no – and, in fact, it often happened that he slipped up and appointed someone as chairman of a committee who had already been appointed as chairman of some other committee. They tried to point this out, but he simply said they must have bad memories. You can’t argue with a king.”

“Well how did Count Abel manage it?”

Mrs Schröder drew her chair closer to the fire and she had a look on her face as if she was telling one of the great ancient myths, like the Greek legends. It was, indeed, the

greatest story of the Infins and it had been told and retold throughout the land.

“Well you see, it was one day many years ago when somebody got the brilliant idea of forming an Everybody-except-the-King Committee, thinking they could plot in his absence. But he simply appointed himself as the external chairman so, although he was technically not part of the committee, he could still be present at its meetings. Of course this meant that he couldn’t be chairman of any other committee.”

“It was then that Count Abel hit upon the wonderful scheme. He called a meeting of the Everyone-except-the-King Committee. Being the external chairman of this committee the King had to be present. So there they all were – the entire infinite population of Infinland.”

“ ‘Oh noble, King, Lord of Committees’, said Count Abel. ‘You have the royal privilege of choosing chairmen for every committee that can come together in Infinland. You can therefore tell us who among us are external chairmen. Tell us now, who are the external chairmen of Infinland.’ ”



“The King was not at all put out by this request. He merely consulted his large book and pointed to the host of Infins on his right and said ‘you my people on my right, and me of course, are the external chairmen of this kingdom’.”

“Well, there were many internal chairmen sitting on the right and many external chairmen who weren’t. Countless Infins knew that he was cheating, but how do you prove a king wrong, especially as no Infin committee ever kept minutes of their meetings. It would be simply *their* word against the King’s.”

“Count Abel then requested that the internal chairmen, and the non chairmen leave the vast hall in which they were meeting. Soon only the external chairmen were left. This included the king, as chairman of the Everyone-except-the-king Committee. Count Abel also remained, because King Aleph had appointed him as external chairman of the Empty Committee. He thought it would be demeaning for Count Abel to be chairman of such an unimportant committee.”

“ ‘Oh King, Lord of Committees, I humbly request that you appoint a chairman for this committee – a chairman for the External Chairmen Committee.’ It was at this point the king knew that he had been outsmarted. He made out that he was consulting his book that was supposed to contain a list of every possible committee, together with the name of their chairman. It was later discovered that the book was entirely blank but at the time it was natural to explain the delay by the fact that it was an infinite list that he was supposed to be consulting. But all this time he was trying to think.”

“You see he was in a dilemma. If he chose somebody present he would, of course, be an *internal* chairman. But the fact that that person was present in the External Chairmen Committee meant that he was also an external chairman – and he wasn’t allowed to be both.”

At this Emily, who is very quick at understanding things of a logical nature, called out with an expression on her face that is usually associated with the word ‘eureka’. “Of course, and if he chose someone who had left, they would, by definition, be an external chairman – but all the external chairmen were supposed to be present. There’s a fundamental contradiction, if we assume that the King did what he claimed to be able to do.”

Brother Charles couldn’t help himself drawing out the mathematical moral from this story. “What the King was supposed to do was logically impossible. There are more Infin committees than there are Infins themselves.”

“But there are infinitely many of each,” protested Alison.

“And therefore there must be bigger and bigger infinite numbers. The number of possible Infin committees is a bigger infinity than the number of Infins themselves.”

“Quite right,” said Mrs Schröder. “King Aleph Zero realised that he had failed and he fled and was never seen again. The Infin Revolution began at that moment. There were elections, and of course, Count Abel was elected king. He became king Aleph the First. But all this was a long time ago. The current king is Aleph the forty-second.”

This is how he signs his name:

$\aleph_{42}$

It’s bigger than $\aleph_{41}$, which is bigger than $\aleph_{40}$ and so on.

“I remember hearing about the great Infin coup from my mother, who heard it from her mother. By the way, my

sister should be home soon. She lives with us and helps with my Albert.”

They spent a very pleasant time, exploring the differences between life in London in 2006 and 1933, and life in Infinland in whatever year it was by their calendar. Indeed it is true that travel broadens the mind.

After a half an hour a lady came into the room, accompanied by a little boy.

“Hullo, Prim. Good evening, Albert. How was church?”

“Oh, boring as hell,” said the little boy. “The sermon went on and on. I thought it would never end.”

“If you talk like that you will *indeed* go to hell, and be bored as well as being roasted over the burning coals,” said his aunt. “Actually the service was very uplifting. But sadly, the young curate announced that he’s leaving next month. He’s moving to a new church further out on the western part of the East-West Road.”

“I s’pose that’ll mean that all the churches ’ll be movin’ further out to make space for his,” said Ivy.

“Oh no, it’s to be a swap of pulpits. Old Reverend Wardrobe is moving here and Branderham, the young curate, is taking his place out there.”

“That will be sad for you,” said Mrs Schröder. I later learnt that the departing curate was a favourite of her sister.

“Oh, I’m sorry,” she continued, “I should have introduced my sister, Primrose Bernstein and my son Albert. She’s taken it into her head to teach him some religion so she drags him off to church. I let her do it, even though Albert

doesn't like it, because I don't think a bit of holiness does anyone any harm. Besides it might make him less of a little devil at home. The cat hasn't been the same ever since he tied a rocket to her and tried to put her into orbit."

And she proceeded to introduce the four travellers from the other world. Or should I say, the other worlds, for the world of London in 2006 was quite a different world to London seventy years earlier.

"The evening service was most wonderful. On the one hand the preacher's prayers were deeply spiritual and on the other hand his voice, when he led the singing, was like an angel's. It was one of the most holy services I have ever attended."

"My sister is a very devout person," explained Mrs Schröder. "She hopes that one day young Albert will appreciate religion as much as she does."

"'ts oright," he said, "but I'd much rather play football than sing hymns."

"Would you four like to stay for dinner? Perhaps you might like to stay the night before continuing on your long journey."

"That's very kind of you," said Brother Charles. "We're not quite sure where we're journeying to but, wherever it is, it's bound to be a long way."

"Specially in a world like this un where everythin' goes on f'rever," said Mrs Dunn.

"Primrose, could you please make up the beds in four of the spare rooms?"

"I bet there are infinitely many spare rooms and if they were already full you could make room for more by getting

everyone to move up.” It was Emily who made this observation, but Mrs Schröder didn’t hear, or if she did hear she ignored the comment, so they never found out how many spare rooms there were.”

They ate dinner and stayed the night very comfortably. Mrs Schröder and Miss Bernstein saw to that.

They were awakened the next morning by the sound of children’s voices – lots of children. And then there was a bell. It sounded as though they were next door to a school, and indeed they were. Alison was curious to see what schools in Infinland were like and whether they were very different to schools in England.

After breakfast she asked Miss Bernstein whether Albert went to that school and whether it would be possible to see the school in action. The answer to each question was a “yes”.

So they were introduced to the headmistress who was impressed by the fact that they had come from far away and so thought that they must be very important visitors.

“I’ll show you a cardinal arithmetic class,” she said. They went into a large room where a sea of faces was chanting the times tables in unison.

Once infinity is infinity.

Twice infinity is infinity.

Three infinities is infinity.

As the chanting continued Emily leaned across to Alison and whispered loudly, “it’s pretty simple arithmetic they have in Infinland”.

The headmistress overheard and said, “oh we do the more advanced arithmetic in senior school. There they learn about the higher infinities.”

“What about the Continuum Hypothesis?” asked Brother Charles who seemed to be the only one who knew what she was talking about.

“They learn about that in Religion. You can’t prove or disprove the Continuum Hypothesis, which is why it has to be taken on faith, along with all the other axioms that make up our creed.”

Primrose Bernstein, who was with them, said “the creed is the basis of our faith and it guides everything we do. Other things being equal it is the most important thing in the world. Without it everything would be meaningless.”

By this time the class had finished their times table. How they said infinitely many things in the space of a couple of minutes is a mystery, but everything in Infinland is deeply mysterious to me.

“Well,” said Mrs Dunn, “we’d better be a gettin’ back.”

“Back where?” asked Alison.

“Back to Upminster o’ course. Don’t you remember we were all on the District Line and our train done got upset? I have to be ’ome to get dinner ready.”

“But that was yesterday. Or was it the day before? We’ll never get back for tea on the day we started,” said Emily.

“We will if we get a move on. Now how do we get back to where we were?”

“Perhaps this might help,” said the headmistress as she held out a bottle marked “DRINK ME PLEASE”.

“That’s the most polite DRINK ME bottle I’ve seen. But do you think it’s safe to try?”

“As far as I know the whole of this infinite land that we find ourselves in is just a little speck of dust in our world. We’re here inside Infinland because we drank from a DRINK ME bottle that made us infinitesimal. Now we can’t get any smaller so my bet is that this DRINK ME bottle will send us back to our normal size. We’ll pop out of that speck of dust and be back in one of the Paradox castles.”

“I thought they were all the same castle,” said Brother Charles.

“Same or different doesn’t matter. We’ll be back in our own world and then all we have to do is to find some way of getting to Upminster.”

So one by one they drank from the DRINK ME bottle and, one by one, they disappeared and, one by one, they reappeared, standing on a railway station. By some miracle it was Upminster station, the very end of the District Line.

How this miracle came about is easy to explain. You see the speck of dust that was Infinland got caught up by a strong gust of wind that blew into Paradox Castle (these old castles are very draughty) and was blown all the way to Upminster and happened to fall in with other specks of dust on Upminster station.

So here they all were all four of them. Wait a minute, let me count again. One, two, three, four, *five*. We seem to have picked up an extra member of our little group. It’s Primrose Bernstein!

“I thought I’d come with you,” she said, “if that’s all right. Now that Reverend Branderham is moving there’s nothing to keep me any more in Infinland. They say that travel can cure a broken heart.”

“But couldn’t you just follow him to his new church?”

“Nicholas has spoken. If he loved me he would never have moved. I must learn to live without him.”

Primrose had been a little bit in love with the Reverend Branderham and the fact that he consented to being sent to a distant parish meant that he didn’t feel the same about her as she did about him.

“Now let’s go,” said Ivy Dunn impatiently. “St Hubert’s not far from here and I’m dyin’ for a cup o’ tea.”

Alison looked at her. “So you’re going to the same monastery as Brother Charles.”

“‘Course I am. I’m the cook at St Huberts ain’t I?”

“Then you know Brother Charles. I thought you said ...”

“Never mind what I said. Sayin’s one think and doin’s another and goin’s yet another think again and if we don’t get agoin’ I’ll never be there to cook dinner.”

Alison found everything very perplexing. In this strange world, every time you thought you understood something the rug was pulled from under you and you went head over heels. All this made her very tired and all she cared about was getting to the monastery. So the five travellers set off and in no time at all ... Don’t you think it very silly when people say “in no time at all,” or “I won’t be a second” or something like that. Things *do* take time to happen and

there's no point in denying it. So I should have said "before you can say 'Schröder-Bernstein' a thousand times, they reached the gates of St Huberts.

Mrs Dunn put on her apron and, before you could say 'Ivy Dunn' several thousand times, they all sat down to a simple but satisfying meal with all the other brothers.

St Huberts was a monastery that was built on the site of an old Saxon priory near Priory Lake in Upminster. It was an open order, governed with a very gentle hand and, though the brothers had taken the vow of poverty, they ate really well. They had a well-stocked farm, a kitchen garden that grew a wide variety of fruit and vegetables, and the nearby lake provided them with fish on holy days. Saint Hubert was the patron saint of mathematics, and many of the brothers were devoted to the study of this noble branch of learning.

PRIM AND THE PRIME MINISTER

As you can see from the above, Primrose Bernstein will be playing an important role in this chapter so I'd better tell you something about her before we begin.

Prim was about 35 years old and was never married. In fact she had never even had a boyfriend. She was very proper in the way she lived and she was devoted to the saint St Emily de Vialar, the patron saint of single women.

She lived with her sister and helped look after her sister's boy, Albert. Over the last ten years she had become increasingly religious and had gone to church once a day and twice on Sundays. Her faithfulness was due to her deep religious conviction, though no doubt it had also been influenced by the fact that she had developed a 'crush' on Nicholas Branderham, the minister.

He delivered the most excruciatingly long sermons, but Prim hung on his every word. His sermons were set out under numbered headings – "Zerothly, I wish to point out ... and firstly we must remember that ... There were infinitely many headings so that you would never know when he was nearing the end. In fact if it wasn't for the fact that in Infinland you can do infinitely many things in finite time his sermons would never have ended.

One memorable sermon was based on the text whereby we are exhorted to forgive, not just seven times, but *seventy* times seven. Now, of course, in the Bible the message was that forgiveness should be without limit. But Branderham interpreted it to mean that the four-hundred and ninety-first sin was unforgiveable.

He also believed that each act of forgiveness had to be for a different sin and so he preached on each sin at some length.

He had intended to keep going beyond 490 and he had invented infinitely many sins to preach about. However when he reached sin number 491 a man stood up and accused him of having the 491st sin – the unpardonable sin of having preached for too long. Fighting broke out and the infinitely long sermon had to be aborted.

Even in our world it is sometimes possible to do infinitely many things in a finite time. Achilles, a very fast runner, challenged a tortoise to a race. He allowed the tortoise a head start of 100 meters. Suppose that each of them starts running at the same time. After some finite time, Achilles will have run 100 meters, bringing him to the tortoise's starting point.

During this time, the tortoise has run a much shorter distance, say 2 meters. It will then take Achilles some further time to run that distance, by which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead.

Thus, whenever Achilles arrives somewhere the tortoise has been, he still has some distance to go before he can even reach the tortoise. A great Greek philosopher, Zeno, argued that Achilles will *never* catch up to the tortoise.

The fallacy in this paradox is the assumption that it is impossible to do infinitely many things in a finite time. Yet if these ‘things’ are Achilles reaching the point where the tortoise had been before, it is perfectly possible. You don’t need to go to Infinland to do this!

So now that Primrose was in a different world, almost the first thing she did was to enquire where she could find the nearest church. Of course she was inside a monastery, where worship was conducted no less than four times a day and you would think that would have satisfied her search for spirituality. But probably at the back of her mind was the fact that all the monks had taken a vow of celibacy, which meant that they couldn't marry, or even have a girlfriend.

Now Prim was convinced that she had a calling. God was calling her to become a minister's wife. That was her vocation. She had been in training for ten years and all she needed now was a husband with the title 'Reverend'. She was convinced that it was a truth universally acknowledged that every single minister with ten years experience in the church must be in want of a wife.

The nearest church to the monastery was St Joseph's in Sunnyside Gardens, about two miles away and Prim was determined to go there on Sunday. It was Church of England which meant that the priest, if not already married, was a potential clerical husband.

She thought it would be good if she had something interesting to say in the religious line to her PCH (potential clerical husband), so she asked Brother Charles to give her a brief outline of the life and work of St Hubert. He told her that St Hubert was patron saint of mathematicians and goldsmiths. She thought this was an odd combination, but saints did have rather diverse interests.

She then asked Brother Charles about the sort of mathematics that he was personally interested in. She didn't understand very much of what he said, but she came away with the impression that it had to do with somebody's prime numbers.

"Prime numbers are some of the most amazing things that God created. They are numbers that can't be split up as the product of two smaller numbers and they have the most wonderful properties."

"Yes I know. God created the primes and one can see God by contemplating them. There are infinitely many of them, from one to eternity, and only an infinite being could have created them," said Primrose, showing that she knew a lot about primes.

"I agree with you Primrose, but we start the primes with the number two. You see we exclude the number one, so two is the smallest prime."

"But the number one can't be split up as the product of two smaller numbers."

"Yes, I agree, but for technical reasons we build that into our definition. A prime number is a number bigger than one that can't be split up ..."

"But that seems rather arbitrary. Why not insist that prime numbers had to be bigger than a hundred. I never realised that mathematicians were so picky!"

Brother Charles seemed uncomfortable. It did seem somewhat arbitrary to exclude the number one. Yet he knew that if you didn't, it would complicate things when it came to proving things.

“Mathematicians are free to define things however they like, and they always have good reasons for defining things the way they do,” was the best he could come up with. He could have explained why excluding the number one was the ‘right’ definition but he didn’t think Primrose would be able to follow.

Alison couldn’t see what prime numbers had to do with religion but Brother Charles seemed to believe that they had and, as Brother Charles knew a lot about both religion and mathematics, she was sure that he must be right. She thought the fact that the Trinity is a prime number must have something to do with it.

As the weeks went by, Primrose attended St Josephs religiously every Sunday. One Sunday Alison and Ivy decided to go with her to find out what she found so attractive in the church. Emily decided to stay behind in the Monastery. She loved walking around the gardens, talking mathematics with Brother Charles.

The church building was certainly very beautiful, with its old stone tower, and the bells were ringing Grandsire Triples. They were early so they stood outside for a few minutes admiring the architecture.

“Look at that Ivy growing up the wall,” said Alison.

Ivy Dunn glared at her but said nothing.

“Oh, I’m sorry. I wasn’t making a joke about your name.”

Ivy’s face relaxed. “That’s all right luv.”

They went inside. The service was very traditional, with four hymns, lots of prayers and a sermon that was about how one could find God in mathematics. If Emily had been there she would have noticed that all the hymn numbers, displayed on the board at the front of the church, were prime numbers, but neither Alison nor Primrose noticed this. Nor did Ivy, but then she still didn't understand what a prime number is.

They sat with Primrose near the front. She sang louder and prayed more fervently than anyone else and by the look on her face you'd think she could see the angels.

As the service came to a close the pure voices of the boy sopranos floated up to the lofty recesses of the church, "... primes without end ... a-men". The service was over.

Prim turned to Alison and said, "Isn't he a dream – those eyes!"

But Alison said, "I was more interested in his sermon. It seemed very persuasive but I'm sorry, Prim, I'm still an agnostic. I just can't believe in your doctrine of the Infinitude of Primes. I mean, perhaps it is true that there are infinitely many prime numbers. But I can't see how you could ever know for sure".

"But Alison, you can see here in *Primes Ancient and Modern* and here in *The Book of Common Primes* that there are primes forever and ever into eternity. Look there's no sign of them petering out."

They had reached the church door and the young Peter McFermatt held out his hand.

“I trust you enjoyed the service, Miss Bernstein?” He greeted them, while looking at Prim with his penetrating blue eyes.

“Oh yes,” gushed Prim, “I found the primes so inspiring. But I’m afraid my friend here is an unbeliever”.

Alison smiled sheepishly. “It’s just that I can’t see how you can be so certain. I admit that it seems very unlikely that the list of primes will ever come to an abrupt end but ... I mean ... it *is* possible. After all primes become rarer as you go among the larger numbers. Is it inconceivable that they eventually die out altogether?”

She pointed vaguely in the direction of the churchyard, but there were too many people behind them waiting to shake the curate’s hand to continue the conversation.

“How about if you and Miss Bernstein come to the rectory this afternoon? We could talk some more over tea and scones.”

Reverend McFermatt poured the tea and passed round the excellent scones that Mrs Duffy had made.

“I’m sure Alison would like to believe that there are infinitely many primes but she doesn’t seem to have enough faith.”

“If only there was some way you could *prove* it to me,” sighed Alison, “but of course that’s impossible. Even if I spent from now till the end of the world factorising numbers I’d only be considering a *finite* number of possibilities. There’s no way the question can ever be settled.”

“Well,” said the minister, “may I call you Alison?”

“If you like.”

“And Miss Bernstein, may I have the privilege of calling you Prim?”

“I’d be delighted if you would, but only if you let me call you Peter.”

“By all means. Actually I was christened Pierre, but Peter will do. Now, Alison, you *do* believe that there are infinitely many numbers altogether don’t you?”

“Oh yes, of course, that’s obvious. I mean you just keep on adding one to get bigger and bigger numbers.”

“And if I claimed that there was a biggest number?” he asked.

“Then I’d say ‘what about that number plus one?’ ”

“Exactly. I’d be forced to admit that my claim was false.”

“But that wouldn’t work for prime numbers,” protested Alison, “because all primes are odd ... except for the number two of course. So the largest-prime-plus-one would be an even number so it couldn’t be prime. And the next number after that mightn’t be prime either.”

At this Reverend McFermatt took a handful of cubes from the sugar bowl and laid them neatly in a row on the damask tablecloth.



“Suppose,” he said, “that each of these sugar cubes represents a prime number. Here’s two and three, five, seven, eleven and thirteen, seventeen, nineteen and twenty-three. Now just suppose,

for argument's sake, that there *does* exist a largest prime."

He scooped up the glistening white cubes and put them back into the sugar bowl. "Just suppose that this bowl contains every prime number up to the largest prime."

"Well, alright then," agreed Alison, "just for the sake of argument. But don't forget that I maintain that believing in a largest prime is just as illogical as believing in the Infinitude of Primes. You'd need infinite time to prove it one way or the other."

"I hope it won't take *that* long," he said looking at his watch and smiling, "I have to conduct Evensong at six o'clock!"

He picked up the pot containing all-the-primes-in-the-world and said, looking earnestly at Prim, "we have here every prime number that exists and, we're supposing, there are only finitely many of them."

"But a very large finite number," said Alison helpfully.

"Now we can multiply all these numbers together to get an exceedingly large number."

"What if there's not enough paper in the whole world to write it down?" she asked.

"That's of no consequence," he assured her, "we can conceive and discuss numbers bigger than the number of atoms in the cosmos. Don't forget, a number's existence doesn't depend on the vital statistics of our universe."

"But I don't see what you're getting at," said Alison as she took another scone. "The result of multiplying all the prime numbers won't be a prime number itself, so where's the contradiction, if there is one?"

“But would you agree that this product-of-all-primes will be divisible, exactly, by all prime numbers?”

“Yes Alison, don’t you see,” said Prim excitedly, “every prime number will go into it exactly because every prime will be one of its factors!” Alison did see. She was more concerned about where the argument would go from there. “Well the product-of-all-primes will be divisible by all primes so the product-of-all-primes plus one can’t be divisible by *any* prime.” Reverend McFermatt leaned over towards Prim in case she was in any doubt.

“You mean because no two consecutive numbers can have a common factor?” said Alison thoughtfully.



“Exactly. So we’re brought to a number which has *no* prime factors. Now this product-of-all-primes-plus-one is bigger than all the ones in the pot, so it’s not a prime itself. But every

number, if not prime itself, can be factorised into prime factors, so it *must* be divisible by at least one prime and hence we reach a contradiction. And remember Alison that this contradiction only came about because we were foolish enough to contemplate a largest prime.”

Alison appeared to recover quickly from the shock of the contradiction, if she felt it at all. But Prim’s face appeared

to be flushed and so the Reverend slid his hand across the table to reassure her. Perhaps Alison was not the only doubter.

Prim had always believed in the Infinitude of Primes because she had always been told to believe it. But now that it was being *proved* she began to have serious doubts because she found it hard to follow the proof. She was so deep in thought as the Reverend McFermatt steadied her arm that she quite forgot for a moment that she even *had* an arm. She screwed up her nose, trying to make sense of it all. At last she discovered the arm, drew it away from the young curate's grasp and picked up an unused sugar cube which had lain unnoticed on her saucer.

"Well all that means," she said, "is that since it isn't divisible by any of the primes already in our pot it must itself be prime – one we overlooked. So we just pop this extra prime into the pot." And she dropped the sugar cube into the bowl.

"But," said Reverend McFermatt, "you'll just get the same contradiction all over again."

Prim picked up a handful of the cubes and dropped them one by one back into the bowl. "So, as fast as you keep getting a contradiction I just keep adding more and more primes to the pot. I can always keep one step ahead of you." She grinned, confident that she had him beaten.

But Reverend McFermatt yielded no ground. In fact he must have been about to deliver another intellectual earthquake because he felt the need to steady her arm again. "The point is Prim, you agreed that we had *all* primes in our

finite pot, and now that I contradict you, you want to add another. That's hardly fair."

But Alison came to her rescue. "Is it such a sin to change one's mind?"

"Look if it were a game of chess I'd be only too glad to let her change her mind to correct an oversight. But she can't claim to be always one step ahead of me just because I let her keep changing her move every time she lands in trouble. Besides, finding a prime that's not in the prime pot isn't an oversight. We agreed to *define* the contents of the pot to consist of *all* primes. It's just an inescapable contradiction. And any assumption that leads to a contradiction must be false. Q.E.D."

He appeared to think that this final blow would require a little extra support and it didn't matter that he upset the sugar bowl in the process because those little crystal cubes had served their purpose. Prim displayed her discomfort at her intellectual position by blushing brightly. She looked down at the spilt cubes on the table as if pleading with them to deliver her from the inescapable conclusion of the argument.

At last she looked up into Reverend McFermatt's deep blue eyes and sighed, "I suppose you're right."

FOOD FOR THE MASSES

Our three friends enjoyed their time at St Hubert's. The weather was, by now, quite warm and they spent a lot of their days out of doors. Emily had made friends with Isaac, the young gardener, and when she wasn't talking mathematics with Brother Charles she was talking about plants and flowers with Isaac.

Now Isaac was not only a skilled gardener, he also knew a lot about the heavens. Often, at night, he would take Emily and Alison out into the garden and they would look up and marvel at what they saw. It seemed that Isaac knew every star by name as if it was a personal friend. He pointed out that most of the stars moved together as if they were painted on a large canvas, while a few wandered all over the place, seemingly at random. This, he explained, was because they were planets, rotating around our sun, while the stars were suns themselves with their own planets.

One afternoon Emily, Alison and Primrose were walking in the garden. Brother Charles was helping out in the library and Mrs Dunn was peeling potatoes in the kitchen. They came across Isaac, sitting under an apple tree. He was probably thinking about the way the planets move, a subject that he often spoke about.



Just then they all heard a voice coming from above them. “EAT ME,” it said. They looked up into the tree, but could see nothing. Alison thought she could make out a pair of eyes and a grin, but it was just a trick of the light. Perhaps it was a bird – a talking bird. Nothing surprised Alison any more.

Just then an apple fell from the tree and was caught by Isaac. “EAT ME,” it said. Now Alison and Emily had been thinking that perhaps they ought to be getting back to their parents in their terrace house in Leinster Gardens. Primrose was beginning to think that life in Infinland might be better after all. Despite some early promising signs, Reverend McFermatt seemed to be more interested in church affairs than affairs of the heart. Besides, who was looking after Albert’s spiritual welfare in her absence? They all knew that by eating something that said “EAT ME” they were bound to be transported somewhere else. That’s sometimes how it works. Of course it might not be where they wanted to go, or even worse it might be Upminster in the time of the dinosaurs. But it was worth taking a chance.

Of course there were Brother Charles and Mrs Dunn to consider. They were home already and would probably not be interested in further travel. But it would be rude to go without saying goodbye.

“EAT ME,” came the voice again. They all agreed that if they didn’t seize the opportunity now it might not come again. They asked Isaac if he would say goodbye to Brother

Charles and Ivy on their behalf but he said that he was coming too.

So one by one they took a bite of the apple and threw it to the next person before they disappeared. Isaac sang softly, with a mouth half full of apple, “we’ll meet again, don’t know where, don’t know when ...”

The next thing they knew they were on a balcony high up on a high-rise apartment block. When I say ‘they’, there were only three of them.

“Where’s Prim?” asked Alison.

“She probably managed to get back to Infinland.” said Charles.

As they looked out they could see that they were in a large modern city. The view consisted of a forest of tall buildings. It was clearly a modern city, but not London. Perhaps they were in Asia or the United States of America.

But Emily caught sight of Big Ben, or at least the tower that houses the Big Ben bell. It was completely dwarfed by the tall buildings around it. “So this is London, but we must be in the future.”

“It could be Las Vegas. I think they have a copy of Big Ben there, not far from the replicas of the pyramids of Egypt.”

Isaac was puzzled. “Surely it can’t be London. The London I come from doesn’t have such strange large trees growing all over the place.”

“But those aren’t trees,” said Alison. They’re buildings, called skyscrapers.”



“It’s a wonder they don’t fall over.”

Just then a lady, wearing strange metallic clothing, came out onto the balcony. “Oh, visitors. Where have you just flown in from?” Alison got such a shock that she dropped the apple.

“We were recently in Upminster and we ate this apple and

The lady gave a squeal of astonishment as she bent down to pick up the apple. It had flown with them from Upminster, and across who knows how many centuries. It was none the worse for wear, except for four bites that had been taken out of it.

“I’ve only seen these in pictures. They stopped making them hundreds of years ago and I’ve never actually seen one. Who makes them?”

Alison and Emily looked at each other. “Why, I suppose God makes them,” said Alison.

“Or Mother Nature,” added Emily.

It was clear that the lady with the silver costume had never heard of either God or Mother Nature.

“What is this God Corporation? Does it trade under the brand name ‘Mother Nature’?”

“Why God made the whole world and every living thing. He’s what you call a Creator,” said Alison.

“You mean to say that he has a monopoly? I knew it would come to this with multinational companies taking over the world.”

“Do you mean to say you’ve never tasted an apple?”

“No, what does it taste like? Look at the shape. I’ve never seen any food so beautifully round.” She took the apple and caressed it.

“Would you like to take a bite?” asked Alison.

“Do you think that would be a good idea?” cautioned Isaac. “Who knows where or when she might get transported to.”

Emily refrained from mentioning prepositions at the end of sentences. Instead she said “I think it would be safe. The apple has stopped saying ‘EAT ME’.”

In fact the apple had been quiet ever since it arrived on the balcony. Perhaps it was now just an ordinary apple.

The lady in the silver costume was acting nervously as if she was trying to decide whether to eat the apple. “I’ve been told that it’s bad to eat apples, which is why they stopped manufacturing them – but it looks so inviting.”

“If you think you shouldn’t, then don’t,” said Isaac.

But she decided to taste the apple anyway. The temptation was too strong. She took a bite and ... nothing happened, except for a look of delight on her face. “I’ve never tasted anything so sweet or crunchy!”

“I suppose we should introduce ourselves. I’m Alison, this is my sister Emily and this is our friend Isaac.”

“I’m so glad you could pop in. I’m Mrs Smith, Evelyn Smith. Let’s go in and I’ll fetch some lunch.”

They went inside the apartment. It was a sitting room, though the chairs were of a modern design and looked as though they were not meant to be sat on. The walls were decorated with abstract paintings that slowly dissolved into others as if they were pictures on a screen.

“Now we only have two dining rooms so I can’t feed you all at once.”

“But can’t we eat together? It would be much more fun,” said Alison innocently.

Mrs Smith’s face showed great shock at this suggestion.

“But my dear, we can’t do that. It’s not proper. I don’t know what the custom is where you come from but *here* we eat alone. I mean eating is such a personal and intimate activity. The only person I could share a meal with would be my husband and by the look of you two girls you’re too young to have husbands.”

The three travellers couldn’t understand what the problem was. “You mean to say that you never have feasts here?” asked Isaac.

“Goodness no. You might be used to having feasts and orgies where you come from but I couldn’t allow anything of the kind taking place in *my* house. No, I suggest you two girls begin. Isaac you won’t mind waiting ten minutes would you?” So, with their eyes still wide open in amazement the two girls were ushered down the hall.

“Alison, you can go into this room and your sister can eat next door. I’ll bring the food along presently.”

The food was packaged in clear plastic bags. Each bag contained rectangular blocks of some unidentifiable substance and these were in a variety of sizes and colours. Each bag had the word “VISITOR” on them, and in smaller print, the words ‘Non-programmed food’.

“What does ‘Non-programmed food’ mean?” asked Emily.

“As you know all food comes from the factory with the name of the person who it’s for, on the plastic wrapper.”

“No, I didn’t know that. You mean it’s like prescription medicine?”

“I’m not sure what you mean. It’s programmed by the food programming team to benefit that particular person. You’re not supposed to eat anyone else’s food. But to cater for visitors we get extra non-programmed food. I’m sorry that it’s nowhere near as nice as the apple you brought with you but I hope you enjoy it all the same.” Mrs Smith then went next door and gave Emily the same non-programmed food.

She was right when she said that it wasn’t as nice as the apple, or indeed any other food Alison had ever eaten. It was mushy and stodgy and tasteless. True, it looked pretty, with all the different colours, but as food it was horrible. Alison did her best to finish it so as not to appear rude. Next door Emily was facing the same problem but her solution was not to eat anything at all.

After a few minutes Mrs Smith knocked on Alison's door. "Have you finished eating?" When Alison said that she had, Mrs Smith came in.

"I'm sorry I couldn't eat it all," she apologised. "I wasn't very hungry. But please tell me why are apples no longer available?"

"It was considered that apples were the source of all the evil in the world. There's a story about the beginning of the world where an apple said "EAT ME" to the first humans and it caused no end of trouble."

"Then there once was a princess, called Princess White, who was given a poisoned apple. She fell asleep for a hundred years. And there was that brilliant programmer who died in the same way. We are taught that apples are all poisonous."

"So why did you eat our apple?"

"Well I thought if it came from another world it might be alright. And it looked so good that I was tempted into eating it. And I'm glad I did – it was delicious!"

Mrs Smith then went next door and knocked. Emily said to come in. She wasn't as apologetic as Alison for not having eaten much – in fact she had eaten nothing at all. She gave back the food unopened and said that she was not going to eat anything that looked like plasticine.

Mrs Smith took them back to the sitting room and then took Isaac off to eat his 'lunch'.

"So everyone here has to eat programmed food?" asked Emily.

"Everyone, except the programmers. They have a different diet, but I'm not sure what it is."

“I’d like to meet a programmer,” said Emily who was quite good with computers.

“I might be able to arrange that. The man next door has a second cousin whose uncle plays sport with the father of a programmer.”

Mrs Smith sent a text message on her smart phone and a few minutes later came a reply. If they would wait downstairs at the network node they would be escorted to the local Food Programming Factory.

The ‘network node’ was like a small railway station in the basement of the building. Before long a tiny train, shaped like a bubble, came noiselessly into the station. The bubble only had four seats, and one was already occupied by a man dressed in a shiny gold suit. The three of them got in, and after some introductions they moved off at high speed.

Isaac sat in the back with Emily. “What did you think of the food?” he whispered to Emily.

“I didn’t think it was edible so I just left it.”

“I ate one mouthful of the bright green stuff, but I had to spit it out.”

“I wonder why they don’t seem to get any *real* food.”

The bubble train soon reached its destination and the man in the gold suit introduced himself. “I’m Dr Tran and I’m to be your guide today.”

“I’m Emily, and this is my sister Alison. And this is Isaac. What’s your Christian name?”

“I don’t know what you mean by ‘Christian’, but my filename is ‘Four’. You see we programmers are called after the counting numbers.”

He escorted them along many corridors and through numerous security doors until they reached a large laboratory. Well, I call it a laboratory because that is what it said on the door. But there were no test tubes or retorts. Just rows and rows of computer screens with what must have been programmers sitting in front of them.

“Do you mind if I ask you why your food here is so terrible,” asked Emily untactfully. “How can you eat the stuff?”

“Oh no, we don’t have to eat it because we’re programmers. Let me take you to the staff canteen.”

He took them down an escalator to a large hall, that looked for all the world like an Elizabethan banqueting hall. The tables groaned with the most wonderful food you have ever seen.

“This is more like it,” said Isaac. “It’s like the royal feasts that were held under the merry King Cole.”

On each table was a suckling pig with an apple in its mouth. Alison was puzzled. Apples were banned in this country she thought. She asked Dr Tran about it.

“Well it *is* banned for the ordinary people. You see apples promote free will and free will allows people to succumb to temptation. Do you know the story of Adam and Eve in your country?”

“Yes, Eve gave an apple to her husband even though they were told not to eat apples.”

“Precisely. And do you know what sort of tree they grew on?”

“I think it was called the Tree of Knowledge of Good and Evil,” said Alison.

“That’s right. Before this neither Adam nor Eve had any knowledge of what was good and what was bad. They had been programmed by the Great Programmer to always choose the good. But there was a bug in the program. Now there was a hacker, called Serpent, and once he discovered the flaw in the program he created a virus to exploit it. This allowed Eve to be persuaded to tempt Adam. Once the two of them ate the apple (because you don’t think Eve would let Adam eat the apple without having a go herself) they suddenly had free will. Not only did they now know the difference between good and evil, they had the freedom to choose between them.”

“So the ordinary people here don’t have free will?”

“Not if we can help it. We program their food so that they behave in a way that we think they should. We’ve just about eliminated free will from the population.”

“But you haven’t eliminated it completely?”

“Well we do occasionally slip up. I mean if the Great Programmer accidentally left a window of opportunity for Serpent to tempt Eve then you can’t blame us if sometimes we make a mistake. But if we come across disobedience we eliminate it as quickly as we can.”

“So the reason you eat normal un-programmed food is ...”

“Well it should be obvious. If we programmers didn’t know the difference between good and evil how could we possibly program the masses to do the right thing? After all, do you believe that the Great Programmer has free will?”

Emily became very pensive. “Yes I suppose the idea of a programmed programmer programming some food for the

population is almost a contradiction. Only someone with free will can choose to eliminate free will from others.”

But Alison and Isaac felt that the conversation was becoming too deep so they were only too happy when Dr Tran took them downstairs to the food processing area. Here raw materials were brought in from outside. Dr Tran was a bit vague when they asked him what the ingredients were. Then the colours were added and finally the mixture went through a special oven where the program was beamed into the mixture using special rays. The mixture was then formed into bricks and baked and packaged and labelled with a particular person’s name.

“Now I want to show you our Programmers’ Training School,” said Dr Tran. “We have to go to the 117th floor.” He took them up in a VFE. In case you don’t know, that stands for Very Fast Elevator. No, I didn’t know what it was either and I had to Google it. You don’t press any buttons – you just tell it where you want to go.

The classroom they were taken into was rather old-fashioned, even by the standards of 2006. It had a very long blackboard at the front and wooden benches and stools. Nobody else was there so Alison, Emily and Isaac sat up at the back. Dr Tran explained why the classroom was so old-fashioned.

“We tried to recreate the style of a classroom of five hundred years ago. Research has shown that it aids creativity. Naturally we want our programmers to be creative. Everyone else has to make do with mere hologram technology and sit on anti-gravitational fields.”

“So you *do* have forests in this land?” asked Isaac, admiring the timber desks. “Do you have any apple trees?”

“You know that apples have been banned for a long time because of their detrimental effects on the citizens. And forests I believe were destroyed along with the dinosaurs.”

“Well, where did you get the timber from?” he asked.

“From where did ...” Emily started to correct his grammar but Dr Tran interrupted. “A long time ago we found some things in a seed bank that looked like hand-grenades, tucked away and forgotten. We set up a large hydroponics factory and grew some trees, Orc trees’ I think they’re called.”

“Could they be oak trees?” asked Isaac.

“They might be. Anyway the scientists tested pieces of the plastic they’re made of, which we call ‘wood’, and found it to be stronger than Ultra Plastic. The problem was to cut it up in the right shapes to make stools and benches. Our usual laser cutter technology just set the wood on fire so our smart engineers developed a special tool called a ‘serrated separator’. You use it in a reciprocal motion, back and forth.”

“I think he means that they cut the wood with a saw,” whispered Emily to Alison. “They’ve had to rediscover our industrial methods.”

At this moment there began to be some noise at the door, as the pupils filed in and took their places on the stools. They were rather young to be potential programmers, probably about Emily’s age, or even younger. There was a lot of noise and then a bell rang and, as a teacher walked in, the class fell silent.

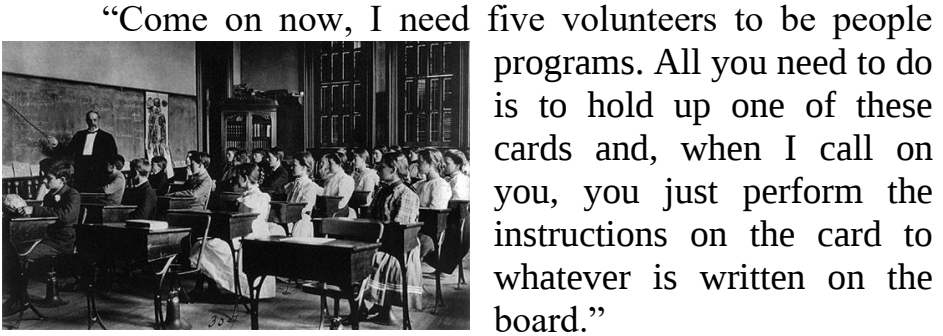
“That’s Professor Turini. He takes the class on the Theory of Computation,” whispered Dr Tran as he sat down with them at the back of the room. “Today’s class is on the Halting Problem.”

“What’s that?” asked Alison.

“Shh! He’s starting his workshop. Just watch.”

WILL IT HALT?

Alison thought it strange that in such a modern world they should be using such old fashioned teaching methods such as ‘chalk and talk’. Professor Turini was standing out the front, asking for volunteers.



“Come on now, I need five volunteers to be people programs. All you need to do is to hold up one of these cards and, when I call on you, you just perform the instructions on the card to whatever is written on the board.”

Emily put up her hand but Alison pulled it down again. “We’re only visitors here,” she whispered.

Noel, in the third row responded enthusiastically to the invitation. “I’ll have a go but I’m not very good at this sort of thing. I’m sure I’ll get it all back-to-front.”

“That’s exactly what I want you to do. Your program is called REVERSE.” He handed Noel a card on which is written the words:

REVERSE

Reverse whatever’s on the board.

“Now whenever I call on you, all you have to do is to erase whatever is on the board and replace it by the same thing backwards.”

“You mean, like in a mirror? So, if you write ‘**pool**’ I write ‘**looq**’.

“No, you keep the letters the right way round but reverse the order of the letters. So the REVERSE of “**pool**’ is ‘**loop**’. Now then, do I have another volunteer?”

Peter nearly fell off his stool with excitement. “If it’s as easy as that then I’m your man and as my mum always says if you want someone to do a job properly and not give up half-way through then ask me because I’m your man and as my mum always says ...”

“I’m sure you are, Peter. Your program is called REPEAT.” He handed him a second card bearing the instruction:

REPEAT

If what is written on the board
ends in ‘T’ put another ‘T’ at the
end of it.

Sitting behind Peter were the Bubble Twins, June and Jane. They called out together “We’d like to help too, but only if we can do it together.”

“Oh, then you’ll like your job.” He gave June Bubble a card on which is written:

DOUBLE

Make a second copy of what’s on
the board, separated by a space

“When I call on you, all you have to do is to make a second copy of whatever appears on the blackboard.”

Jane said to her sister, “Ooh, I’ll do the copying because I’ve got the steadier hand. You can hold up the instructions in case I forget them.”

“Right, let’s practise those three programs.”

But Mary, who was sitting in the front row, and had her hand up all the time, called out, “What about me? I knew you’d forget me. It’s just not fair!”

“You’ll get your chance, Miss Contrary, I’ve got just the job for you. But we’ll just practice these first three. Now when I call out the name of your program you have to perform the instructions on your card to whatever is on the blackboard. If I say REVERSE that’s your cue, Leon.”

“Do you mean me?” asked Noel.

“Sorry, Noel, yes it’s you I mean. And if I say REPEAT its over to you Peter. And your cue, girls, is DOUBLE.”

He wrote the letters RAH on the board.

“OK it’s DOUBLE first.”

The Bubble sisters wrote a second RAH next to the first to get RAH RAH. Professor Turini then said, “Now REVERSE”.

Noel rubbed out the message RAH RAH and replaced it with HAR HAR. “Now,” said Professor Turini. “DOUBLE again”. The message now became HAR HAR HAR HAR.

“And finally REPEAT.” Peter was about to start tacking a row of R’s on the end of the data but the Prof caught him just in time. “No Pete. Your instructions are to add T’s and *only* when what is already there ends in T. When it ends in anything else you do nothing.” Peter, somewhat disappointed, sat down again.

“Now we’ll try another one.” He cleaned the board and wrote the word EXIT and said, “REVERSE”. Noel changed EXIT into TIXE.

Peter had a confused look on his face. “Isn’t ENTRANCE the reverse of EXIT?”

“No Pete, Noel’s right. I said REVERSE, not OPPOSITE. OK, now DOUBLE.” Jane Bubble added a second TIXE.

“REVERSE.” Noel replaced the TIXE TIXE with EXIT EXIT.

“And now REPEAT.” Peter excitedly wrote a long sequence of T’s, getting

EXIT EXITTTTTTTTTTTTTTTTTT.....

until he ran out of blackboard. The Prof had to restrain him from continuing across the wall.

“That’s stupid!” said Mary. “Whenever Pete takes off nobody else can follow him.”

“No, Mary, it’s not stupid. It’s just what happens when a computer program crashes because it gets into a loop.”

“Well it’s stupid to ever get into a loop. The computer should be clever enough to know that it’s being told to get into a never-ending loop and escape out the offending program.”

“But Mary, it’s not always so easy to ensure that a program will go on forever.”

“Course it is! Any fool could see what was going to happen when Pete took over. A clever computer would be able to examine any programs it had to run and refuse those which would make it crash.”

“But that would need *another* program to work out what would happen.”

“So what! It might be a complicated program but I’m sure someone smart like Tim could come up with one. You just get Tim’s program to look at the one you’re going to run and if it’s OK it rings a bell and if it would loop forever it rings a buzzer. Then you’d know not to let the computer run any program that sets off the buzzer.”

“But this program would have to be able to work on every possible program.”

“Sure, and what’s wrong with that?”

“Well, it would even have to be able to work on itself.”

“Well any dum dum can see that Tim’s program would always halt so if you ran it on itself you’d get the bell, of course. Now when are you going to give me my program, or had you forgotten?”

“OK Mary Contrary, I’ve got just the program for you. It’s called DISOBEY.” He gave her a card with the following instructions:

DISOBEY

If what’s written on the board is HALT then

REPEAT.

If it is LOOP then REVERSE.

Otherwise print “?”

“But that’s silly. If I read HALT I have to tell Peter to REPEAT and he goes on forever writing
HALTTTTTTT.....

and if I read LOOP I tell Noel to REVERSE and he replaces LOOP by POOL and then halts. So whatever I'm told to do I'll be doing the opposite!"

"That's why it's called DISOBEY, Miss Contrary! Let's try it now." He wrote POOL on the board. "Now REVERSE." Noel changed it to LOOP.

"And now DISOBEY."

Mary thought aloud. "Well since it says LOOP I have to REVERSE it again" She picked up the duster but the professor gently restrained her.

"What's the matter, I've got to do a REVERSE, don't I?"

"Not you, your job is to activate Leon as a subroutine. *He* does the actual reversing."

"Oh, all right then. Go on Noel. (I suppose that's who you meant.) Noel reversed LOOP and once again the word POOL was written on the board.

"You're all doing very well." He cleaned the board and wrote HALT. "OK Mary DISOBEY". Mary gave Peter a hard thump and Peter started writing dozens of T's until the Professor took hold of his arm to break him out of his infinite loop.

"Now has it ever occurred to you that a program can be made to operate on itself?"

Tim thought for a moment and then said, "Well I suppose I could write a program called COUNT which counts the number of letters in a piece of text and I could run it on a copy of the COUNT program itself."

“Exactly, so if you ran COUNT on Alice in Wonderland what would you get?”

“Just a moment,” said Peter, “I’ve got a copy here. Do I count the spaces?”

“No, just the letters and punctuation marks.”

Peter’s finger flew across the pages and the pages fluttered as he turned them over. “The answer is 115,950 more or less,” he said after a few minutes.

“So June, if DOUBLE acted upon itself, what would happen?”

“DOUBLE DOUBLE toil and trouble – well just DOUBLE DOUBLE I suppose.”

“Now, Leon, what if you carry out REVERSE on the REVERSE program?” Noel had given up making the professor get his name right. “You’d get ESREVER.”

“Pete, would you mind doing REPEAT on REPEAT.”

“What do you mean?”

Professor Turini wrote REPEAT on the board. Now Pete, carry out the REPEAT program on it.” He thought for a minute, then he picked up the chalk and turned it into

REPEATTTTTTTTTTTTTT.....

“So if DOUBLE acts upon itself it will halt. The same is true of REVERSE. But if REPEAT acts on itself, as data, it will never halt. Now Tim, the last program is yours. It’s called PREDICT.”

“I knew you’d say something like that. You’re going to tell me that my program predicts whether or not any program will halt, or whether it will go into an infinite loop.

“Exactly, and because the answer will depend on what data it’s given it needs to be given both the program and the data.” He handed Tim the last card with the program:

PREDICT

If the program will halt when given itself as
the data,
print out HALT
but if the program will loop, print out LOOP

“That’s not very difficult,” said Noel. “All Tim’s program has to do is just run the given program and if it halts then it prints out HALT and if it doesn’t halt ...”

The Prof saw where he was going and finished his sentence.

“... then you’d never be able to break into it to print out the message LOOP”

“Well can’t you just break it out of its loop if it seems to be going on too long?” asked Peter.

“How long is too long? A program might take a very long time and still halt. Even if you waited a hundred years you wouldn’t know for certain that it’s not going to halt at some time in the future.”

“Well how’s Tim going to do it?” demanded Noel.

“He can’t. It’s impossible.”

But Mary couldn’t accept that. “That’s rubbish. Tim’s a computer whiz. And even if Tim can’t, someone will one day be able to do it. It makes me mad when people say that something is impossible just because they’re not clever enough to do it themselves! Someone clever can examine the

program and work out whether it will halt, without actually running it.”

“Well, we’re supposing for the sake of argument that Tim has done it and PREDICT is that program. I’m now going to give myself the last program. It’s called BANG.” He held up a card displaying the four words:

BANG DOUBLE PREDICT DISOBEY
--

“What do you think will happen if we feed BANG into itself? Will it halt or will it go on forever?”

June said that she thought it would halt but Jane thought it wouldn’t. They didn’t often disagree with one another so it was going to be interesting to see who was right.

“Let’s suppose Jane is correct.”

“Of course I am. You don’t have to suppose.” Professor Turini wrote the word BANG on the board. “Now which of you Bubbles is going to DOUBLE?”

“It’s my turn,” said Jane as she wrote a second BANG. The board now contained the words BANG BANG.

“Now Tim, PREDICT.”

“How can I? I don’t know whether BANG will halt when given itself as data.”

“But we’re *assuming* that Jane is correct and she says it will halt.”

“Well, then I suppose I rub it out and write HALT.” He did this.

“Now Mary, it’s over to you.”

She gave Peter a nudge and said, “it’s my job to activate REPEAT.” So Peter wrote as many T’s after HALT as he could fit on the blackboard.

“So Jane was wrong. If BANG halts then it doesn’t.”

“See,” said June triumphantly. “I was right and you were wrong!”

Professor Turini erased the board and again wrote the word BANG. “This time we’ll assume that June is right.”

“Which I am – we just proved it!”

“So June, you get to DOUBLE this time.” June wrote a second BANG and again the words BANG BANG appeared on the board.

“Tim, you now have to PREDICT.”

“Well, since we’re assuming that June is right this time I have to say that it *won’t* halt. But I have to halt after writing my prediction.” And he erased the BANG BANG and wrote LOOP.

“Now Mary, DISOBEY.”

Mary said, “since my input is LOOP I have to activate REVERSE. She tapped Noel on the shoulder and he wrote POOL and there the program stopped.

“This time we were assuming that June was correct and that BANG wouldn’t halt, and yet it *has*. So June was wrong as well.”

Mary was getting very irritated. “That’s the dumbest thing I ever heard. One of them *has* to be correct. Yet you’re saying they’re *both* wrong. That’s stupid!”

“But don’t forget our first assumption. The one we’ve been assuming all along.”

“What’s that?” asked Tim.

“We assumed that such a program as PREDICT *does* exist. Our only possible conclusion is that it *doesn’t*.”

“But you gave me the program PREDICT. Here are the instructions.” And he held up his card.

“The trouble with that is that it says “*if* the program will halt, but it doesn’t tell you exactly *how* you can find out whether it will. The fact that PREDICT leads to a contradiction means there’s no possible way of always determining whether a program will halt. That’s the Halting Problem.”

Mary started to walk out of the classroom. “My Halting Problem is the fact that this stupid lesson seems to be going on forever. Tim, do you predict it will ever HALT?” At that moment the end-of-lesson bell was heard and Tim replied, “indeed I do.”

It only took a minute for the class to rush out. Dr Tran led the others to the lifts. As the door opened Dr Tran said, “you’ll have to go by yourselves – just call out where you want to go.”

“What floor do we say, for the exit?”

“If you want to you can just say Mrs Smith’s apartment. The lift travels horizontally as well as vertically.”

“But what if it takes us to the wrong Mrs Smith? ‘Smith’ is such a common name.”

“Oh, it’s context sensitive. It does a brain scan and knows which Mrs Smith you’re staying with.”

The doors closed. Emily said, “take us to Upminster.” As the lift descended Alison said, “what if it takes us to the wrong Upminster. There might be one in the United States.”

“Context sensitive,” reminded Emily.

By now they were at ground level and it seemed to be moving sideways at a very high speed.

“But what if it takes us there in the wrong year? It might be Upminster in this God-forsaken time in the future. Anyway, do we want to go back to 2006 or 1933?”

“Context sensitive! The last time we were in Upminster it was 1933.”

It seemed to take longer than it did on the District Line back in 1933, but if the truth be told it was only about ten minutes. Ten minutes on a train, when you can look out of the window, is no time at all. But ten minutes in a lift is a lifetime. As time went on the lift seemed to get narrower and narrower. By the time the doors eventually opened they were crushed together into quite a small space. They were relieved to be out and, looking back, they found that the lift was just one of the confessionals in the chapel of St Huberts. They were home at last – well temporarily so for Alison and Emily. But was it 1933? Indeed it was – in fact it was only a few minutes after the moment when they had eaten the apple. They found Mrs Dunn in the kitchen, assisted by Primrose.

“There you are Isaac! I’ve been looking for you everywhere. Where have you been? I need a bushel of apples for the pies I’m baking for tomorrow’s banquet.”

“But Prim, I thought you’d gone back to Infinland. When you ate the apple, and you weren’t with us in that horrid place, I presumed you’d somehow gone back home.”

“I took a bite, but I spat it out. It tastes quite different to Infin apples.”

“Any way it’s good to see you. Charles, what’s this banquet you’re having tomorrow?”

“Brother Bertrand,” replied Brother Charles looking up from his work, “has found a most ingenious counter-example to a very famous conjecture.”

“What’s a conserjuncture?” asked Ivy. Is it some sort of jam?”

“No,” replied Emily, “a conjecture is a guess that something is always true. But a counterexample is an example that shows that it can sometimes be false.” She’d heard her father talk about these things.

“I don’t get it,” replied Ivy, still puzzled. “Is a counter-sample what you get when you turns the hands of a clock back’ards?”

“No, that’s ‘counter clockwise’”. Emily loved explaining things that she understood well. “Just suppose you were a missionary and you went to some far away island. You might conjecture that the natives would always be friendly. Captain Cook made such a conjecture when he went to one of the Hawaiian Islands. He then discovered a counter-example when he got himself killed!”

“In mathematics a conjecture is really just a guess,” added Brother Charles. “Someone might be able to prove that it’s always true. On the other hand someone might come up with a counter-example. But the funny thing is that often

when someone makes a conjecture that sounds likely, and no-one seems to be able to work out whether it's true or not, people pat him on the back and make a lot of fuss of them. But if, many years later, someone else either proves that it's true, or else finds a counterexample, they never seem to be as famous as the person who made the lucky guess. Have you ever heard of Pierre Fermat?"

"Not Peter McFermatt?," said Primrose, her eyes lighting up. "He's the minister at St Joseph's."

"No, said Brother Charles. "Pierre Fermat lived well over two hundred years ago."

"I suppose he was French," said Ivy. "Was he a famous chef?"

"I haven't heard of him either," said Alison.

But Emily knew about him. "Don't you remember Daddy talked about Fermat's Last Theorem."

"Just so," said Brother Charles. "But really, it was Fermat's Last Lucky Guess! If anyone ever succeeds in proving it ..."

"Or finds a counter-sample," interjected Ivy.

"... I'm sure that he won't become as famous as old Pierre," continued Brother Charles.

"But it *has* been proved," said Emily. "I remember Daddy telling us that somebody by the name of Andrew Wiles proved it a few years ago."

"Really," said Brother Charles. "I never heard that."

"Just goes to show that you was right," said Ivy laughing. "This Fermat fellow just made a guess and apparently everyone's 'eard of 'im – 'cepting Alison and me,

that is. But this wily fellow is a nobody, even though he actually done proved it.”

“You’re right Ivy,” said Emily. “They still call it Fermat’s Last Theorem, not Wiles’ First Theorem.”

“I can’t believe it’s finally been proved, after two hundred and fifty years!”

“It’s almost *three* hundred and fifty years. Oh, I forgot, this world we’re in is seventy years ago. In our world it was proved about ten years ago. You’ve got another sixty years to wait.”

“Did this Fermat fellow tell anybody about his conjecture?”

“Well, he claimed to have actually had a proof. But he wrote in the margin of one of his books that, although he had a really marvellous proof, the margin wasn’t big enough to write it out.”

“What a cop out. I bet he was a masematickle cowboy – always making conjectitures and sayin’ he’s got wunerfull proofs which he never really done have!” said Ivy.

But Brother Charles came to Fermat’s defence. “Actually he was a very clever mathematician – one of the best. He had a marvellous nose for mathematical truth. He *did* prove lots of things, and most things he conjectured were proved eventually. It’s amazing that it should have taken these hundreds of years to be proved. Goodness me, what a shock – Fermat’s Last Theorem has been proved at last! Wait till I tell the other brothers.”

“Well technically, it’s been proved in *my* world.” said Emily. “In your world you still have to wait another sixty years.”

“So what was Bertrand’s counterexample?”

“Oh, it’s too complicated to explain in five minutes. But he comes from a family of counter-examplists. His cousin, also called Bertrand, once shook the foundations of mathematics with a famous paradox, that was in effect, a counter-example to something that everyone thought was obviously true.”

THE BEAVER AT THE BANQUET

The next day was to be a great celebration. There was a lot of hustle and bustle as preparations were made for a grand banquet. Brother Charles was in the Great Hall, polishing the silver, along with several of his brother Brothers. Ivy and Alison were giving him a hand. Emily was looking on.

Ivy handed some silver knives and forks to Emily while Alison polished some candlesticks. “So, Brother Bertrand has found a counterexample to some famous conjecture. I suppose it’ll be in all the newspapers – or at least it will be hot news in all the mathematical departments around the country.”

“I doubt it.” sniffed Brother Charles. I don’t think he had a high opinion of newspaper men. “Nobody ever seems to make a fuss when someone finds a counter-example. I don’t know why. Newspapers are like that. If someone blows up a bridge – that’s big news. But if I choose *not* to blow up a bridge, which is just as important, nobody is the slightest bit interested in hearing about it. However here at St Huberts a counter-example is considered to be just as good as a theorem.”

He went on polishing while Ivy and Alison went off to the kitchen. Emily, having laid out the silver on the long tables, just sat and watched.

That afternoon was warm and sunny. The sweet smell of the flowers drifted on the gentle breeze. It was the sort of weather when it was a crime to be indoors. Alison and Emily went out into the gardens to speak with Isaac. He was

working in an area where a most unusual type of flower grew. They had long stems, without any leaves, surmounted by a single large, white, flower.

Now I know you're expecting these flowers to be unusual because they had faces, and were singing one of those songs that such singing flowers usually sing. But these flowers weren't unusual in that way. What was curious was that the stalk, instead of going straight up, was knotted in some way. Moreover each one seemed to have a different type of knot.

"What do you call these?" asked Emily.

"These, why they be Ashley's Knot-flowers," answered Isaac.

"But why do they grow in that knotted fashion?"

"I suppose it's because the opening bud follows the sun. It goes up and down, and sideways, and sometimes grows up through a loop to make a knot. There are many varieties and each one in the same variety grows the same knot. You can tell what perfume the flower has just by examining the knot. The trouble is that sometimes a knot will look different to the one next to it but it's really the same knot."

"How can you tell whether two Knot-flowers are the same variety?"

"From their scent."

"But can you tell from the shape of the knot?"

"Well, that puzzled me for years, but young Alex here has worked out a method. He's one of the under-gardeners – bright lad, that. You see you draw a picture, and join up both ends – the top of the stalk with the bottom, just like a knotted

piece of rope if you tied the two ends together. Then you do some simple arithmetic and out comes a number. If two Knot-flowers have different numbers they're different varieties."

"So you just draw a picture of the stem and count the number of crossovers," suggested Emily.

"No, it's more complicated than that. If you took your joined-up piece of rope you could scrunch it up and change the number of crossings but the knot would still be the same. So two Knot-flowers could be of the same variety and yet their stems could look quite different – even with a different number of crossings."

"And if two Knot-flowers have the same number they're the same variety?" suggested Alison.

"Not exactly – it only works one way. There are different varieties with the same number. Alex is working on that. He thinks he's come up with something better than a number."

Isaac got out a notebook from his pocket and drew one of the Knot-flowers. He showed Emily and Alison how to calculate its number. It turned out to be five.

"You try this one," he said to Emily. She drew a sketch and, joining both ends, she followed Isaac's instructions.

"I think this one is a three. Do you always get an odd number?"

"That certainly is the case. So because the first one is a five, and the second a three, they *must* be different varieties. Here, smell them. See how different their scent is."

"Why only odd numbers? And can you get one with the number one?"

“It would take me too long to explain to you why you can’t get even numbers. And no, you can’t get the number one – not with a Knot-flower at least. But if you take an ordinary unknotted flower, where the stalk grows straight, you’ll get a circle when you join the ends together. Then, if you do the calculations you get the number one.”

Emily was thinking deeply “So if you get the number one you’d know that it was not a Knot-Flower, but just an ordinary white flower that someone had knotted up?”

“I suppose so,” said Isaac.

Just then Alexander had come up and had heard Isaac’s last remark. “No sir, remember last month we found a very highly knotted Knot-flower. Its number came out as one but the new method I’ve been working on showed that it was different to an unknotted circle. If you made a model of that stem out of rope and joined the ends together you’d not be able to untangle it without cutting it open. So, two Knot-flowers *can* have the same number without necessarily being the same species.”

That evening the banqueting hall of the Priory was full of excited guests. It was a warm evening and so the windows were open. The smell of the flowers outside wafted in. The brothers were excited because of Brother Bertrand’s accomplishment, and excited in anticipation of all the wonderful food that was to be served. Alison and Emily, too, were looking forward to it. They had never been to such a feast.

There were at least a dozen ‘removes’. That’s what you might call ‘courses’. First was the turtle soup. Then some goose pâté on crisp rye biscuits. Then the fish dish. The fish had come from Priory Lake. I don’t know what sort of fish they were, but they came in whole, on silver platters, with their bulging eyes looking up at the hungry faces.

And so it went on. The monks at St Huberts might have taken the vow of poverty but no-one on the kitchen staff seems to have heard of it. After the wild boar and pheasant, and many more meat dishes, came the desserts. There was a wonderful syllabub, and dates drizzled with almond milk and honey.

“What’s this,” asked Emily, sampling a most delicious dessert.

“Why that’s one of Ivy’s specialities. We call it ‘Wardonys in Syryp’. It’s pears boiled in a



sauce made up of cinnamon, red wine and sugar – plus a few other special ingredients. It used to be quite popular in the middle ages.”

The meal was accompanied by some beautiful music on the harp, played by Brother George. Brother George was an expert on the mathematical aspects of harmony and Brother Charles explained that the music he played illustrated some of his theories.

After the desserts Brother Bertrand, the guest of honour, stood up and held up three small identical wooden

boxes. "I need a volunteer. Alison was sitting nearby so she offered her services.

"Now one of these boxes contains two gold sovereigns. One contains a sovereign and a piece of silver with the same size and weight as a sovereign. The third box has two of these pieces of silver. I want you, young lady, to choose a box."

Alison chose one of the boxes. "Now, with your eyes closed, I want you to select one of the two coins."

Alison closed her eyes and reached into the box and pulled out a coin. It was a gold sovereign. She then opened her eyes and held up the gold coin. Everyone cheered.

"Now I'd like the other young lady to tell me what are the chances of the other coin in this box being a silver coin."

Emily thought she knew the answer. "Surely it's 50-50. The box must be either the double gold or the gold and silver one. There's a 50-50 chance of each. If it's the gold and silver box that Alison chose then it would be certain that the other one is silver. But if it's the all gold box then there's no chance of silver. So it's a 50-50 chance that the other is also silver."

"So, if I asked you to give me a pound and promised that if it is indeed silver I'd give you back two pound ten shillings, you'd agree?"

"Of course I would. On average I'd get back one pound five shillings."

So Alison reached into the box and pulled out ... another gold coin. "Oh, well, I was just unlucky."

"No my dear, the odds were against you. In fact you only had one chance in three of winning."

"But surely ...".

“For you to have won, the box your sister was holding must have been the one with mixed coins. It couldn’t have been the one with two silver coins, and if it was the one with two gold coins you would have lost.”

“Well, yes.”

“And there was only one chance in three of her having chosen the mixed box. If she had you would be certain to win. But that only gives you one chance in three of winning?”

“I suppose. But what if she had chosen a silver coin?”

“Then I would have asked you to bet on the remaining coin in that box being gold. Either way you would have only had one chance in three of winning. On average, out of three goes you would have wagered three pounds and got back two pounds ten shillings – a loss of ten shillings. That’s an average loss of three shillings and four-pence each time!”

Just then Emily noticed a disturbance at the far end of the table. There was the most unusual creature she had ever seen. It was brown – a bit like a large furry rat. But it had a large, broad, tail and stood upright on its back, webbed, feet – probably it was two feet tall. The most unusual thing about it was that it wore a bright red costume rather like you see in pictures, being worn by musketeers. He had a magnificent white lace collar, and wore a sword at his side. What was even more unusual was that it spoke in a heavy French accent.

“What on earth is that?” Alison asked Brother Charles, pointing to the far end of the hall.

“Oh, that’s Galois,” he replied. “He’s come over from French Canada. I think they call them beavers over there.”

Now the name Galois is probably one you’ve never heard before, and as it’s difficult to pronounce I want to make sure that you can say it correctly. Galois, himself, has a short temper and you might run the risk of having your head cut off if you made it rhyme with ‘toy’.

You pronounce Galois as ‘Galwuh’. Galois is proud of the fact that nothing rhymes with it. That’s why you’ve never come across the name in any of the poetry you’ve ever read.

Emily’s face broke into a broad smile as she recognised him. “Why that’s the beaver that spoke to me in London. He told me how to catch up with you – you remember, I told you about him.”

Galois was surrounded by a group of fellow beavers, all dressed as musketeers. If you don’t know what the word “swagger” means you’d get the general idea by watching Galois. He danced around as if he was the most important person in the world – afraid of no-one.

Just then he jumped up onto the table and drew his sword. “Death to Cardinal Phillipe! ... ”

There was a long hushed silence. “... if he turn traitor.” The monks, who had been sitting quietly enjoying the meal, looked around in terror and several of them even jumped out of an open window.

After a few minutes, Galois got down from the table, sat down again and continued eating his bowl of water-lilies. Peace returned, and Emily asked Brother Charles what was happening. “Who is Cardinal Phillipe?”

“Oh, he’s a high church official who’s coming over here from the Vatican. He’s due tomorrow, though some say that his secret police are already among us. That’s why some of the brothers jumped out of the window. They didn’t want to be mixed up in it if there was any trouble about what Galois said.”

“But why does that creature Galois call him a traitor?”

“Worse than a traitor. It’s said that he denies the Axiom of Choice!”

“What’s that?”

“Oh, it would take too long to explain. I’ll tell you some day. Suffice to say it’s something we all believe in here at St Huberts. It’s the foundation of our faith.”

“Well, I’d like to meet this Galois.” said Emily. “Is he safe?”

“No, he’s not safe – but he *is* good. He’s safe, so long as you’re not a traitor. Yes, come with me.”

They walked to the other end of the banqueting hall, carefully stepping over upturned plates and goblets where some of the brothers had knocked them over in their hurry to jump out of the window. As they came up to the bevy of beavers, they stood aside and Galois stood there in all his glory.

He recognised Emily. “Why, if eet isn’t the *jeune mademoiselle* that I helped in Bayswater.”

“Yes, sir, it is I.” Emily was very particular about her grammar and would never say “it’s me”. “You were in a terrible hurry that day. I didn’t have time to thank you.”

“No matter. I’m always verry busy. I ‘ave so much to do.”

“What *do* you do?”

He answered by taking out a tiny business card from under his cloak and giving it to Alison.

“I’m ze great ‘untsman – quests and hunts.”

Emily didn’t think he looked like a spider. She remembered the huntsman spiders that sometimes invaded her family’s holiday house in Australia.

“And what did you say your name was?”

Alison had been caught out like this before, although, it must be admitted, not by a beaver. “I didn’t say, but now that you ask, my name is Alison.”

“I didn’t ask what your name *is*, I asked what you said your name *was*. Never mind.”

Alison didn’t quite know how to respond to such a prickly creature. “Well, my name has *always* been Alison – at least as long as I can remember. But what hunts and quests do you go on?” She could see Emily out of the corner of her eye so she rephrased the question. “On what quests and hunts do you go?”

“Well, back in Canada, where I come from, I was ‘ired to hunt for ze krans, zat ees ze kransky zauzeges. At last I found ze boucherie zat makes zeze zausesges. Why people need to eat ze meat I can’t zink. Me, I would like ze sausage filled with juicy bark. Well, vat do you zink? When I asked ze boucher vat ‘e put in ‘is kransky zauseges ‘e said ‘e only used beaver meat! I was nearly going to draw my sword, but eensted I backed away and zed I ‘ad made zis mistake. Eet is ze only time I ‘ave run from ze danger.”

“So what are you ... for what now are you questing?”

“I am zearching for ze Ring of Ramanujan.”

“I never heard of it. Is it very valuable?”

“Eet is ze most valuable ring in ze whole world. ‘ooveer wears eet has ze great power.”

“Oh, I know, and when you find it you’ll take it to the Mountain of Doom, and destroy it before someone evil can use it.”

“I never ‘eard of zat mountain. Is eet far from ‘ere?”

“No, it’s just that I’ve heard of legends about powerful rings and how they have to either be destroyed or returned to the river from where they came. How does this ring work?”

Brother Charles had obviously heard of this ring, so he took over the explanation. “It’s a pigeon-blood ruby ring, set in gold. Whoever wears this ring is supposed to be given the power of mathematical intuition. He, or she, can know deep truths in mathematics without the necessity of having to prove them.”

“Perhaps Fermat wore this ring when he wrote down his discovery that took hundreds of years to prove.”

“Eet is posseeble. Certainly Ramanujan wore eet when he made all ’is discoveries.”

“Who was Ramanujan – was he a king?”

“Oh, no. He was a Brahman Indian of the lowest cast. I don’t know how he came by the ring. Perhaps his great grandfather worked for a Sultan. But this ring gave him great insight into mathematics. He was able to state theorems without having proved them. He was largely self-taught but when he came to the attention of some mathematicians in Cambridge they invited him to come to England.”

“And did he do well here?”

“He certainly amazed the professional mathematicians. He could state facts that no-one knew about, and when asked how he knew them he was often at a loss to say how. But sure enough all of his statements were proved to be true – none were ever proved false. He lived in England for five years but the climate and the food didn’t suit him, and he died.”

“So the ring?”

“He always wore the ring and it’s claimed that this is how he came by his remarkable intuition.”

“So where is the ring now?”

Galois interjected, brandishing his sword. “I weesh I knew. It got lost, or was stolen. I have followed up ze leads and zat ’as brought me ’ere to Upmeenster. Death to any rat who gets in my way!”

Alison wondered whether he was talking literally when he said “rat”. If beavers could be on a quest for the missing Ring of Ramanujan then perhaps giant rats might be his adversaries. Or perhaps he was only talking figuratively.

“I ’ave two leads. Zese other beavers will go to a nearby manor ’ouse. But I weesh to enlist you, Mademoiselle Emily, to go *avec moi* to ze *Pie Voleuse* shop in a nearby village.”

“I’m sorry,” said Emily. “I got the bit about going with you, but what was the shop?”

“You know – ze opera by Verdi – *La Gazza Ladra* – ze *Thieving Magpie*. It’s a shop zat sells ze old zings.”

“You mean it’s an antique shop?” said Alison.

“Yes zat ees eet. I ’eard zat zay ’ad zum ruby rings.”

“Yes, I’ll come with you,” said Emily. “You’ll come, won’t you Alison?”

“Of course,” said Alison.

“If it’s adventure to be ’ad I’ll be a coming with you too,” said Ivy. “We’ll take the Priory’s cart. Perhaps I can take the reins.”

“You can come too lady. It’ll be less suspicious to ’ave some of ze fair sex in our party. But I weel do ze driving.”

THE THIEVING MAGPIE

The party set out next morning in an open cart that they had borrowed from the Priory. Galois was driving, and Emily sat next to him. Behind them there were Alison and Mrs Dunn. Ivy had arranged for Primrose to fill in for her in the Priory kitchen. Brother Charles didn't come as he had to meet up with Cardinal Phillipe, and to smooth things over in case the Cardinal got wind of what had happened the night before.

They went at quite a pace as Galois extended the white horse. He was eager to get to the Thieving Magpie before lunch. It was very pleasant moving along the lanes in the warm sunshine, talking about all sorts of things. Ivy talked about the preparation that had gone into last night's banquet. Emily talked about Brother Bertrand's counter-example. Galois told of his recent duels. Alison just wanted to talk about how lovely it was for friends to be enjoying each other's company while going on a pleasant outing.

After some time Ivy asked if she could take over the reins. Galois reluctantly agreed, but he was worried that she might slow them down. However, with Ivy driving, they went even faster and Emily was worried that the cart might turn over. Not much talking was done for the rest of the journey!

All went well and they didn't have a spill. By noon they had pulled up outside The Thieving Magpie.

"Funny name for an antique shop," said Alison. "Makes you wonder how they came by all their stock!"

However the reason for the name was explained when they saw above the door: "Proprietor: Magda Pycroft". They went in and at first they couldn't see any one in attendance.

That was because the shop was packed full of interesting old wares – grandfather clocks and oak desks piled high with chairs and boxes. Then, they saw an old face, bordered by white hair, peeping through the legs of a chair that was sitting on top of a table.

“Can I help you?”

“We’re looking for a ring for my sister,” said Emily.

“Yes, my dear, and what sort of ring did you have in mind?”

Emily looked at Galois. He explained, “it ees a gold ring wiz a ruby stone.”

Mrs Pycroft only had two rings in stock that answered this description and Galois shook his head at both of them. While he was examining them Alison was looking at some other jewellery in a glass case.

“Oh, what a lovely gold locket!” I’ve always wanted a locket like that.”

“Then you shall have that one,” said Galois generously.

“It’s got the cutest gold chain, but it looks a bit weak. I guess I’ll have to replace it with a stronger one. Never mind. I wonder what’s inside. Perhaps a lock of hair, or a photograph of a loved one.”

“I can’t tell you luv, I’ve never been able to get it open.”

“Let me have a go,” said Galois. He held it between his two front paws and, in a moment, it clicked open. “You just have to apply a little pressure here, and here, at the same time. I’ve seen one like it.”

Alison opened it and, to her disappointment, there was



nothing inside it. Nevertheless she still wanted it very much. Perhaps she could put a lock of Emily's hair inside.

"How much is it?" asked Galois, holding out a roll of banknotes.

"Nothing." said Mrs Pyecroft.

"But you can't rightly get somefin' for nuffin'," protested Ivy.

"But you're getting nothing for nothing," explained the shopkeeper. "There's nothing inside the locket. The gold container is just the packaging."

Emily laughed at this explanation. On the one hand it was very logical – on the other it was nonsense. "Oh, well, if that's how it is, we'll take it."

"As long as you realise that I won't be putting it in a silk lined box. What you're buying is 'nothing' wrapped up in a gold package. Now, there's a twelve month warranty on the contents but, as usual, that doesn't cover the packaging."

They continued to browse amongst the wonderful treasures. There was an old fashioned wind-up gramophone, with a large sound horn at the top. What was unusual were three paddles that stuck out from each side. They could be levers to operate the machinery, except they were far too long

for that. It was as if it was a galleon and these paddles were the oars. The cloth on which it was standing was rumpled and looked like the waves, and indeed there was water dripping out of the horn.

Then Alison's eyes were caught by a long cheval mirror. That's a long narrow mirror on a stand that you can tilt, and view your entire length at once.

"Look at this lovely ...," she said and then she jumped a little as if she had had a shock. For, reflected in this mirror, and standing right beside her, and a little behind, was a beautiful red-headed woman, wearing a full light blue dress, trimmed in brown. The skirt of the dress was full as if there was a system of hoops underneath.

Alison looked around to see the woman herself but ... there was nobody there. Now usually when you see an image on the other side of a mirror there is something real on *this* side. Usually when you have a mirror, there are two of everything, one in this world and one in Looking Glass Land.

Well, they say that vampires are never reflected in a mirror, but this was the other way round. You usually expect to be able to see what's producing a reflection.

Alison looked back at the mirror, and then at the space beside her. There was only one red-headed woman to be seen and she was the reflection on the *other* side of the mirror! Alison smiled at her, and the red-headed image smiled back. Alison said "good morning," but the mirror must have been sound-proof because she received no answer. Then the lady in the mirror walked away and disappeared.

“We best be agoin’,” said Ivy. “It must be well past lunch time.” So Alison put on her locket and they proceeded to walk out of the door. Once outside Alison decided to have another look at the things in the window. She was looking intently at an old wooden doll when a big rat, dressed in dirty trousers and a green shirt, ran towards her and snatched the locket, breaking the chain, and ran off.

When I say ‘rat’, I’m speaking figuratively. He had a face that looked a bit like a rat, and rat’s whiskers. But he didn’t have fur all over his body, and he was nearly six feet tall, so I guess he was a man – unlike Galois who was *definitely* a beaver.

Now beavers are not known for being able to move fast on land and when Galois took off after him it seemed as though the thieving ‘rat’ would get away. But fortunately, or unfortunately depending whose side you’re on, the man went into a narrow laneway, not realising it was blocked at the other end. What made it worse, or better depending whose side you’re on, the man tripped over garbage tin and fell to the ground. In a moment the beaver was standing over him with a sword at his throat.

“Sorry,” he whimpered. “I think it must be the wrong locket. I swear that it looked like one my grandmother lost last week.”

Galois didn’t believe his story, and nor do I. But he snatched the locket and let the man go.

Alison and Emily had just caught up with them. “Here’s ze locket,” he said to Alison. I’m afraid the chain somehow got lost but you said you were going to replace eet so I suppose it doesn’t matter.”

Just then Ivy came up, puffing with the exertion of trying to run. “What about lunch?” she said. Galois was eager to get on to plan B. They still hadn’t found the Ring. But he was persuaded to wait, so they walked up the road to the Royal Bull.

Over lunch Galois explained that Brother Charles had told him that at St Huberts the word for a piece of jewellery is ‘axiom’. That was because St Hubert once said that an axiom is like a jewel. You see he was the patron saint of mathematicians and goldsmiths. Axioms are little things in mathematics and jewellery is something that goldsmiths make.

“Axioms – what a funny name for jewellery,” said Ivy. “I never ’eard that word ’efore.”

“You see, in mathematics an axiom is a basic assumption,” explained Emily. You can’t prove something out of nothing. You have to start somewhere.”

This gave Alison an idea. “I suppose it’s a bit like the Apostle’s Creed. ‘I believe in God almighty ...’ An axiom in mathematics must be something you have to believe – in faith.”

“Well, if it’s an axiom, this locket must be the Axiom of the Empty Set,” explained Brother Charles. “This says that ‘nothing’ exists.”

“But how can that be?” said Ivy. “I know that *I* exists.”

“‘Cogito ergo sum’,” said Emily. Now she was really showing off! “It’s Latin for ‘I think, therefore I am.’ You don’t really know if everything you can see or hear is just an illusion but you *do* know of your own existence.”

Brother Charles clarified his explanation. “No, that’s not what I meant. The Axiom of the Empty Set says you can have a set with nothing in it.”

“That’s obvious,” said Alison.

“No, it’s nonsense,” said Ivy. “A set of knives might ‘ave six knives, or even four. But a set of no knives is just plain silly.”

“It might be nonsense,” but you can still have it. An empty box could be a set of no knives.”

“So then it’s obvious,” said Alison.

“But can you *prove* it?” asked her sister.

“Well no, it’s just obvious.”

“There you are then – an axiom is something that’s obviously true to you, but something that you can’t prove.”

“Like you can’t prove that Dad loves us both – but we know it’s true. So that’s an axiom,” suggested Emily.

Ivy was sure they were talking nonsense. “But why all this hoo-ha about nuffin? Who’d waste their time talking about it?”

“Shakespeare did. He wrote *Much Ado About Nothing*. Someone once wrote a whole book about the many aspects of nothing. ‘Nothing’ is a fundamental, and very important, concept in mathematics. In a sense the whole of mathematics can be built up from the empty set.”

“You mean in the same way the world was created from a void?”

“Perhaps.”

“But if this ‘nothing’ ceased to exist would it matter?”

“Well how would you get on with writing numbers if you didn’t have a zero? You have to remember the clumsy

way they wrote numbers up to the middle ages – Roman numerals.”

“I suppose.”

“What other axioms are there?”

“Oh, there are several others which I’ll tell you about some time. But here comes the ploughman’s lunch.”

When they emerged from the Royal Bull, they discovered that their horse and cart were no longer tied up outside.

“Oh, that Rat has stolen our horse.” said Alison indignantly.

“More likely he’s just let him loose. In that case he’ll find his way home by himself. Looks like we’ll have to walk back to St Huberts.” said Galois.

RING OF BRIGHT WATER

Early the next morning Emily, Alison, Ivy and Brother Charles went out fishing in Priory Lake. The water was bright and the sun was glancing off the surface just as if the rays were pebbles that had been thrown.

“Oh how bright the water is today,” murmured Brother Charles.

At this Alison went into a kind of reverie. “Bright water, bright water. ... What does that remind me of?” She reached deep down into her well of memory. “Emily, wasn’t that a book we read? *The Ring of Bright Water*.”

Emily thought for a few moments. “Oh yes, I remember. It was about otters, wasn’t it? Have you read it Brother Charles?”

“Can’t say I have.”

Ivy assured them that she hadn’t heard of it either, but as she read very little, apart from the cooking pages in women’s magazines, that didn’t mean much.

“No wonder Brother Charles hasn’t heard of it, Emily. It hasn’t been written yet. This is 1933, don’t forget.”

It was now Brother Charles’ turn to go into deep thought. “I do remember it, now I come to think of it – not as a book but as a valuable piece of jewellery belonging to a Maharajah. It was a diamond that was called ‘bright water’. I wonder whether it has anything to do with Ramanujan’s ring.”

“But Ramanujan wasn’t a Maharajah – far from it.”

“Yes, but his grandfather had once been a servant in a Maharajah’s palace. It’s just possible that he was given it for

some outstanding service. Was the book associated with any particular place?"

"I think it was the Isle of Skye," suggested Emily.

"Well I think we should go there to see if any of the locals have heard of it."

"Not me," said Ivy. "The last time I've left the cookin' in the 'ands of Miss Primress didn't I get no end of complaints from the brothers."

"I'll ask Galois if he wants to come. Beavers are a bit like otters, so he could prove useful."

It took a couple of days to organise the trip. They went by train, and across to the Kyle of Lochalsh and then by boat across to the island. Being 1933 the bridge connecting the island with the mainland had not yet been built. They landed at Kyleakin and held a council down by the water's edge, to work out a plan. "You go and ask in the village," said Galois, taking the leadership. "I'll go into the water and ask the sea creatures."

So he stripped off his uniform, folded it neatly on the ground and placed his sword on top. The enquiries in the village didn't result in any information so they returned to where they had left Galois. As they came near they heard angry voices. The insults were coming from Galois and a sea otter.

"You rat," spat out the otter.

"Weasel!" cried Galois."

"Vegetarian!" was the next insult coming from the otter. You and I have known some perfectly lovely people who, for some reason or another don't eat meat. But for an

otter who feeds on small rodents, and fish, and even baby beavers, a vegetarian is beneath contempt.



“Cannibal,” said Galois. “I demand satisfaction!” and

he reached for his sword. Now, for Galois, ‘satisfaction’ meant a duel. A time and a place were agreed on and each of the enemies would arrive with their ‘seconds’. These were friends who saw to it that

there was fair play. They also had the gloomy task of removing the body if their friend should lose the contest.

“But duels are outlawed in Scotland. Perhaps we should arrange a truel,” said the otter.

“Never heard of such a thing,” said Galois. “but if it preserves my honour I’m happy to follow local custom. Does a truel require a third person?”

“Oh, no – it’s a cross between a duel and a triathlon. A truel is a fight of honour where there are three contests and the decision is based on the best of three. Galois could see a problem with this. “But eef I win ze first two contests and get killed in ze third I become a posthumous winner. Zat geeves me leetle zatisfaction.”

“Ah, but the contests are non lethal. One of the contests could be a game of chess. That’s usually non lethal, though I do recall somebody once choking on a pawn.”

“And who decides on the contests?” asked Galois, still not sure of the fairness of the arrangement.

The otter pointed to a small stone cottage. “Mr McGregor will oblige.” He waddled over and honked at the

door. A man, unmistakably a Scot, emerged. “Gavin McGregor at your service.”

The otter explained the situation and Mr McGregor announced that the first contest was to be a game of McGregor Says. It turned out to be a variation of the old game, O’Grady Says. The two contestants stand, side by side, facing the front, each with an unloaded pistol. Galois was on the left and the otter was on his right. Mr McGregor would call out orders to the contestants alternately. (I forget how it was decided who should be first.) If a contestant moved ever so slightly when it wasn’t his turn, he was out. And if he failed to obey the instruction when it *was* his turn he was also out.

Now the curious thing was that there were only three commands which could be given, and they could be given in any order. They were LEFT, RIGHT and LOAD.

“I thought these contests were supposed to be non lethal,” objected Galois.

Mr McGregor laughed. “The guns aren’t really loaded. When I give the command LOAD you simply raise your pistol, as if about to fire – that’s if it’s your turn.”

At the instructions LEFT and RIGHT the contestants were expected to do a left or right turn. But whenever the gun was loaded they had to do the *opposite* to the command – turn left instead of right, or vice-versa. And if the gun was loaded, and they were told to LOAD, they had to do the opposite, that is, fire. But all that was required in this case was to call out “BANG!” and drop their arm, with the pistol again by their side.

They started by both facing the water, with the otter on the right side of Galois. The otter was to go first. “LEFT”. The otter turned left. Galois stood still, but out of the corner of his eye he could see the otter looking towards him. “LOAD”. This was Galois’ turn. He raised his pistol. Even though the gun wasn’t really loaded he would have liked to point it towards his opponent, but they were standing side by side and *his* gun was aimed at a tree.

“LOAD.” This was the otter’s turn. He raised his pistol as if about to fire. Galois could see it pointing towards him and it made him uneasy. “RIGHT”. Galois was about to turn right, but remembered in time that his gun was loaded. That meant he had to turn left. He was now looking away from the otter and the otter was facing him with the gun loaded. What if Mr McGregor was wrong and the guns were really loaded!

“LEFT”. Without hesitating the otter turned right, facing the water. “LOAD”. Galois felt silly calling out “Bang!,” as if the pistol was a child’s toy. This wasn’t doing much for his sense of honour. In any case if the guns had contained live ammunition his gun would have fired harmlessly away from the otter.

The contest continued for a few more minutes. Galois, who had trouble knowing his left from his right at the best of times, found himself concentrating really hard.

“RIGHT”. It was Galois’ turn and his gun was loaded. He turned right, forgetting that he was supposed to do the opposite. He had lost the first contest.

The second contest was a board game, a bit like checkers or draughts. Each player had eight large dice – those

of Galois were red and the otter's were green. Each player rolled his dice and placed them successively along their back row. Each of the dice occupied the whole of the square on which they stood and the moves consisted of rolling the dice along an edge into an adjacent square. A piece was captured if the opponent rolled one of his dice next to it, with the same number on top.

Emily watched intently, trying to work out how the game worked. It seemed that each player could make up to four moves at a time and she noticed that frequently Galois used his four moves to roll a die around in a loop, such as forward, right, back and left. This meant that the die ended up where it had started, but had rotated in some way.

Emily was very skilled at manipulating a Rubik's cube and she knew that such moves were important. These were moves like $aba^{-1}b^{-1}$ where a and b were moves and a^{-1} and b^{-1} were the reverse moves.

If a was the act of putting on your socks and b was putting on your shoes then $aba^{-1}b^{-1}$ meant putting on your socks, then putting on your shoes *and then taking off your socks* (quite hard with your shoes still on) and finally taking off your shoes! You see how important the order in which you do things can be.

The game proceeded and the otter appeared to be floundering, losing piece after piece. Eventually Galois won the game. It was one-all. Now for the third and deciding contest.

“The third contest, said Mr McGregor, “is neither a physical nor a mental one. The winner is the one who makes his opponent happiest.”

Galois and the otter stared at each other while they digested the implications of this third trial. Then, at the same moment, they burst out laughing and embraced each other. Their goodwill was genuine – it was not an attempt to win the third contest. They suddenly realised the futility of the whole truel.

“Whew,” said Alison, overcome by the emotion of reconciliation. “It’s like something out of the Sermon on the Mount. Blessed are the peacemakers ...”

By now the beaver and the otter had become the best of friends. “You can call me Oscar,” said the otter. “And you would be ...”

“Galois,” said the beaver, with his paw outstretched.

“You must come for tea. You must all come for tea ... except, I don’t have any twigs or bark – just field mice and rats. You like rats and mice I presume,” he said to Emily and Alison.

Their faces had turned white. They didn’t want to be rude but there were limits. Fortunately Mr McGregor came to the rescue. Why don’t you come to *my* cottage? I’ve got good Scottish food, and the animals can bring their own.”

So Galois got dressed and they went across to the stone cottage. Oscar and Galois disappeared to search for the sort of food that each of them could eat.

“Now girls, what can I offer you?” invited Mr McGregor. “I’ve some wholesome oatmeal gruel. Just the

thing after a truel”. At this he laughed heartily, as if it was the funniest joke ever conceived by man. But Alison and Emily were not laughing. The prospect of oatmeal gruel was a little more welcome than rats and mice, but only just. “Or I might have some left-over haggis.”

“I don’t think I’m very hungry,” said Alison faintly.

“That would be very nice sir,” said Brother Charles in a polite, but unconvincing voice.” Mr McGregor was enjoying their discomfort. “Oh I do have some roast pork and potatoes. Perhaps you’d be wanting that.”

At this their faces lit up and their hunger returned. They all sat around and had a most memorable lunch – not just because of the food but, just like all memorable meals, because of the company.

“I don’t suppose you’ve heard of a ring called the Bright Water Ring?” Oscar said he hadn’t but Mr McGregor said that it was like a name the locals used to call a small bay not far away. It was never called that on the maps, but if you said ‘Ring of Bright Water’ to anyone over sixty they’d know what you meant.”

“So, it’s not a ring that you’d wear on your finger?”

“Not that I know of.”

So the last few days had been a wild goose chase. They were no nearer finding Ramanujan’s ring than they were several days ago. But they had made new friends and had spent the time agreeably, so they didn’t mind.

Two days later, back at St Huberts, they were sitting around after one of Mrs Dunn’s masterpieces.

“So where to from here?” Emily asked Brother Charles. Do you have any fresh leads?” Charles didn’t, but Galois did.

There’s a house not far from St Huberts. I have some information that the two men who live there may be able to shed some light on the mystery. If one can’t the other one might.”

TWO'S COMPANY

They retraced their journey back to the monastery, something that took a couple of days. The day after they got back was Sunday so they all went to church. Primrose stayed behind to talk to Peter McFermatt – except that, unlike the other parishioners, she called him Pierre, and even once it was ‘Pierre dear’. It seems that she was making some progress with him after all.

So it was not until Monday that they set off. Ivy drove them part of the way, but the house they were looking for was in the forest and it was easier to find it on foot.

The weather was quite warm for October. Galois didn't like walking. He was much more at home in the water. They walked a long way through a forest and the autumn leaves crunched under their feet. The trees were still shedding their leaves.

Presently, in the distance and standing next to one of the trees, was a most beautiful red-headed woman wearing a magnificent light blue dress, trimmed in autumn brown. I think you've guessed it. She was the woman that Emily had seen in the cheval mirror. As before, the skirt of the gown was full, supported by a system of hoops.



“I bet she could have hidden her whole family under that dress,” said Alison.

The next moment the front of the dress opened, like the curtains in a theatre, and an identically dressed red-headed woman, less than half her size, peeped out. I say ‘woman’, because though she was the size of a child, she appeared to be fully grown. In fact she seemed to be identical to the first woman, only smaller. Her dress, of blue and brown, was as full as the first.

Then the dress of that smaller lady opened and an even more diminutive red-headed lady was revealed, between her legs. This was repeated many times, and a seemingly infinite sequence of beautiful women was revealed.

“That’s a good example of how an infinity of things can be compressed into a finite space,” observed Emily.

When she caught sight of our adventurers the red-headed lady hid behind a tree and all her miniatures returned to their places underneath her skirt.

Galois called out to her, in a foreign tongue. What he said, the others couldn’t understand – perhaps it was in French. But whatever it was it must have reassured the lady and she moved across slowly towards them. When you have a convergent sequence of smaller ladies under your skirt, it is very difficult to walk!

They met up with our group and the smaller ladies escaped from their hiding place and stood beside the largest one in an ever diminishing row. Now that they could be seen more closely it was clear that there were only twelve ladies in all, not infinitely many.

“I thought you must been from the palace, to fetch me back,” she said in a thick, foreign accent. Clearly she had not come from Buckingham Palace but, which country she was from, the girls could not identify.

“Je suis Princess Sophie, and my father wants me marry my cousin, which I not want. So I flee.”

“And are these other ladies also princesses?” asked Charles.

“Oh no, they are my waiting ladies,” she said. We dress to look same so if king’s men come they won’t know which I am.”

“But won’t they assume that *you* are the biggest one?”

“Oh no, I have bottles.” She showed two bottles, one marked ‘DRINK ME’ and the other ‘DRINK ME’. “If they drink this, they be same size as me and if they drink other one they go back to smaller size.”

She demonstrated by passing the DRINK ME bottle down the row. One by one the ladies in waiting drank, and soon they were all the same size. They were not identical in appearance, but the similarity of their dresses and hair (the others were wearing wigs to match the princess’s colour) meant that it was difficult to tell them apart.

“Can you tell me way to Upminster? I be told that we can asylum there in St Hubert’s monastery.” Of course our friends were the very people to give directions. They explained how to get there and the ladies in waiting each drank from the ‘DRINK ME’ bottle and they were again restored to their diminished sizes. They nested together like a set of Russian dolls under the skirts of the princess and slowly moved off.

Alison, Emily and Galois kept walking and at about four o'clock they found themselves outside a rather quaint thatched house that stood all by itself.

"I don't think I can walk any further," gasped Galois. "My paws are killing me."

"Let's ask the people in the house if we can sit down for a while," suggested Alison. "And maybe they'll give us a cup of water. I'm so thirsty."

They walked up the path to the two front doors. Yes, I said *two* front doors. Most houses only have one.

"Perhaps there are two flats," said Alison. But there was only one number, the number two, on a brass plate right between the two doors. And underneath was just the one button. So they rang.

After a few seconds both doors opened at the same moment and two, rather fat, gentlemen stood – one in each door. They looked exactly the same – just like Tweedledum and Tweedledee. But it couldn't have been them because they're in a different story!

Standing in the two doorways, as they were, they reminded Emily of something she'd seen hanging on her grandmother's wall back in Australia. It was an instrument that told you what the weather was going to be. There was a small wooden house with two doorways. And there were two figures, each standing on a piece of wood that rotated, so that whenever one figure moved outside the other moved inside and vice versa.

One figure had an umbrella and the other was dressed in a bathing costume. If the one with the swimming trunks came out of his door, the other would be inside and the weather was going to be fine. But if the one with the umbrella came out of *his* door, his brother would be sheltering inside and rain was predicted.



“How do you do, young girl. My name is Twodledum. Let me introduce you to my brother Twodledee.”

“Perhaps I’m in Wonderland after all,” thought Alison. Aloud she said, “and this is my sister Emily, and this is our friend Galois.”

“We were wondering if we could have some water. We’ve walked a long way and we’re terribly thirsty.”

“By all means. Won’t you come in?”

“Which door are we to use,” asked Emily very politely.

“This one, of course,” said Twodledum. “This is the entrance door. The other is the exit.”

“But houses where we come from only have *one* door. We go in and out of the same door.” At this Twodledum and Twodledee burst into laughter as if using the same door to both go in and out was the most preposterous thing they’d ever heard of.

So they all went into the ‘in’ door and found themselves in a large living room. At the far end were two identical fireplaces, and on the fireplaces were two identical clocks, both telling exactly the same time. What was curious

about the clocks were the numbers painted on the dial. They were II, IV, VI, VIII, X and, at the top, was XII.

Alison and Emily knew their Roman numerals and wondered why only even numbers were used. Alison asked, “I can’t help noticing your clocks. Why do you only use even numbers?”

“What other numbers are there?” said Twodledum. “I’m sure you children have been taught to count. Two, Four, Six, Eight, Ten and Twelve.”

“Or, counting backwards,” said his brother, “twelve, ten, eight, six, four, two, zero!” At the sound of “zero” a whistle went off in the next room. “Oh, that’ll be the kettle – you’re just in time for tea.”

The living room had two doors, one on each side. Twodledum went out of the door to the left, while Twodledee went out to the right.

Presently the two brothers returned. One was holding two kettles and the other had a plate of biscuits in each hand. The cups and saucers and milk were already on the table, just as if they had been expecting them.



Twodledum and Twodledee swapped a teapot for a plate, so they were each now holding both a teapot and a plate of biscuits. Then they each, simultaneously, poured into the same cup, one from each side. “We

find that it makes a better cup of tea to pour from two teapots,” they explained.

They did this for each cup and repeated the process with two milk jugs. Then they passed around the two plates of biscuits. Alison noticed that the biscuits were identical on the two plates and wondered why they needed to offer both. Still, she was a polite girl and took one biscuit from each plate without saying anything.

They talked of many things. I was going to say ‘of cabbages and kings’, but then I thought that sounded a bit silly. When it was time to go, Twodledum said to Alison, “You’re such a pretty girl, I want to give you a gift. And he presented her with a box, wrapped in gold paper, tied with a pink ribbon.

Just then Twodledee stood up, with a similar box in his hand, except that it was in silver paper and was tied with a blue ribbon. “Ah,” thought Emily, “this’ll be something for me.”

But instead of presenting it to Emily, he went up to Alison and said, “Because you’re so wise, I want to give you this gift.” Emily wasn’t normally jealous of Alison – she loved her big sister. But, nevertheless, a little arrow of jealousy pierced her heart.

Alison opened the first box. Inside, nestling in a bed of pink silk, was a most exquisite gold earring, with a large ruby dangling down. But, it was just the one earring. She had been allowed to have her ears pierced when she was ten so she was able to wear it. “But what am I going to do with just one earring?” she wondered. Then she opened the second box.

She wondered if it was the other earring. But because the one in the gold box was gold she was worried that the one in the silver box would be silver. But it wasn't. It was an exact copy of the first earring. So she had a pair after all.

When Emily saw that it was the other half of the first gift she was a little less jealous. Besides, she'd never had her ears pierced so she wouldn't have been able to wear them anyway. Instead she was happy for Alison. Alison put them on and turned her head from side to side so that everyone could see them sparkle in the firelight.

"I suppose this gift is another axiom," she said to Emily.

"Indeed," said Galois. "It must be ze Axiom of Pairing."

"You mean as in peeling potatoes?" asked Alison.

"No," said Galois. "I mean pairing, as in *deux* zings. If you take any two zings you can make a pair out of zem."

"Like a pair of two earrings," said Alison. "But what if they don't match? You couldn't call them a pair in that case."

Emily said she thought she heard her father talk about this. "Dad says the Axiom of Pairing means you can make a pair out of *any* two things."

"What if they don't match? You can't have a pair of socks if one is red and the other is blue."

"They don't have to match. A pair of odd socks is still a pair. But anyway, you can always find something in common with any two things."

"Or any two people?" asked Alison. "But what do I have in common with Ivy?"

"Well," said Emily, "you're both good at cooking."

“And between Brother Charles and you – I suppose that it’s the fact that you’re both good at mathematics.”

“True, but it doesn’t have to be something you’re both good at. Ivy and I both have names that end in ‘Y’.”

“I see, and Galois and Ivy both love adventure.”

“But so do I.”

“Well they both love going fast in the carriage. You and I don’t.”

“True”.

“Which reminds me,” chipped in Galois. “How are we going to get back to St Huberts? It’s getting dark.”

The brothers had the answer. They each offered a horse and carriage to take them back – the two girls in one and Galois in the other. But as they all wanted to go back together they managed to persuade the brothers to let them take just the one carriage. But, of course, they had to agree to having *two* horses to pull it.

HIGH TIME

They got back tired and hungry to St Huberts, late that night. The Brothers had all gone to bed but Primrose and Ivy between them made a simple meal. Emily was so tired she actually fell asleep over her meal and almost drowned in her soup! But it's marvellous what a good night's sleep can do and when they awoke the next morning, the sun was already high in the sky, and they felt refreshed and ready to go on another adventure.

While they had been away Brother Charles had dealt with Cardinal Phillipe and he and his secret police were on their way back to Rome. Charles had received some information that might help in their search for the Ring of Ramanujan. There was a report that it might have been hidden in a farmhouse near Dungeness in Kent.

That was quite a way from where they were. Ivy decided not to go. She was needed in the kitchen. Apparently the Brothers had complained that the food wasn't as good while Primrose was in charge. But Brother Charles, freed from his responsibilities with the Cardinal, was eager to go. So Alison, Emily, Brother Charles and Galois made up the party.

There was a railway station on the main line to London, not too far from the Twoodle house where they'd been the evening before so they thought they could kill two birds with one stone by returning the horses and carriage and catching the train to London from there. Though why one would want to kill two birds at all is beyond me.

The brothers Twodledum and Twodledee were surprised to see them back so soon, but were glad nevertheless. They tried to persuade them to stay for lunch but our friends said they had a train to catch. So the brothers drove them to the station. This time they got their own way and *two* carriages were employed. Twodledum took the two girls while Twodledee drove the two men – or should I say, one man and a beaver.

They arrived in London and stayed overnight in a monastery that was of the same order as St Huberts. The next day they took the train from Charing Cross to Hythe. There they had planned to catch the miniature train to Dungeness.

But they were annoyed that the Hythe station at which they arrived was some distance out of town, and they had a long walk to reach the station for the Hythe to Dymchurch railway.

The miniature railway had only opened a few years earlier. It ran on a 15 inch track, with railway carriages that are only just big enough to sit in. Alison said that she felt like being Alice after she had become bigger when she drank one of the DRINK ME bottles. But Galois said he was quite comfortable.



The trip to Dungeness was about 13 miles, and it took about an hour. First stop was Dymchurch. Fortunately, for

they were in a hurry, it didn't stop at St Mary's Bay. Then came a few stations whose names were variations on the theme of 'Romney', and finally they arrived in Dungeness.

As they travelled along the sky became covered in black clouds and soon there was a light drizzle. They had not prepared for wet weather, and as the windows could not be closed, they got quite damp.

From the Dungeness station they set off on foot, using a map that they had borrowed from the Priory Library. Their route took them across open ground and it was very muddy underfoot as it had rained very heavily the day before. Fortunately it was only drizzling at the moment.

After some time, Emily ran on ahead. She had consulted the map and thought she knew the way. Soon the ground became very boggy. Emily had just found an open patch at the far end of the wood. The other three were some way behind, still among the trees, so they had lost sight of her. Just then there was a scream!

They ran up to the edge of the forest and saw Emily about twenty metres from the last tree. At least they saw some of her. They saw her head, and shoulders, and two arms floundering around. The rest of her had disappeared into the bog. Brother Charles knelt down on the firm ground, as close as he dared to Emily, but it was no use. He couldn't reach her. Alison found a branch under one of the trees and handed it to Brother Charles. But, alas, he still couldn't reach Emily. And she was slowly sinking further and further into the bog.

Just then Galois started doing something at the base of the nearest tree, at the edge of the bog. He was gnawing away at the tree – which is something that beavers are good at. It

seemed to take an age before the tree started to sway. But Galois will tell you that he'd never in his life gnawed so quickly.

The tree quivered, swayed and then fell with an almighty crash. Galois, with all his experience, had managed to make the tree fall in exactly the right direction. If it had fallen away from the bog it would have been a waste of time, and Emily would have disappeared before they could do anything else. And if it had fallen *towards* Emily, right on her head I hate to think of what would have happened.

As it was, the tree fell close enough to Emily that she could hold onto it while Galois climbed out along the trunk until he could reach her. It was a long and difficult job to get her back to dry land. Well, that's not quite true. Even where they were standing, the land was not dry. But at least it was more or less solid. So they got Emily back to *terra firma*, at last.

They went back through the woods and consulted the map to find a different way of getting to where they wanted to go. They found a track and set off. They hadn't gone very far when the skies opened as if buckets of water were being poured on their heads. They ran on, hoping to find shelter, with the rain dribbling down their necks. Their shoes and socks made slushing sounds with every step.

Presently they saw a fairly large farm house. "Let's try there," said Brother Charles, once again taking the lead. They came up to the door – just one front door this time – and knocked. As they knocked the door creaked open. There was

nobody there. The door just opened with the force of the knock.

They went inside, glad to get out of the pouring rain. As they dribbled from room to room they began to realise that the house was deserted. It wasn't just that somebody had just stepped out to post a letter. (Why would they in all this rain?) No, it was clear that nobody had lived there for some time – that is, excepting the spiders, for there were cobwebs in every room.

They went from room to room, and also upstairs. They even looked in all the cupboards. Alison opened a large chest of drawers and out tumbled two large boxes. She opened one of them and Emily opened the other. Inside each box were two smaller boxes. It was like those Russian dolls, except there were two in each. Each of these smaller boxes contained two even smaller ones.

By now they were really curious to know what the ultimate contents might be, if anything. "Maybe these are just empty sets," said Alison.

As each box contained two smaller boxes, there were soon eight boxes, then sixteen, and finally thirty-two quite small boxes, all over the floor. They busily opened each of these boxes and they were all quite empty – except one of the smallest ones. It contained what looked like a gold pocket watch.

"I know what this is," said Emily. "It's like what Grandpa had – a fob watch. You know, he had a special pocket he put it in and a chain and he'd pull it out with, and then he'd open the front and say 'well children it's time for bed'."

“I wonder if it’s going,” said Emily as she held it up to her ear. “Yes, I can hear it ticking. So somebody must have been here today, or maybe yesterday, to wind it up. And yet the place looks like it hasn’t been lived in for years.”

“Silly Emily, that’s your own watch you can hear.” Indeed, Emily had a wind-up watch she had on her wrist when she had come from the other world. It was quite unusual in 2006 to have a watch that you have to wind up every day, but Emily liked old-fashioned things. Because she’d held the fob watch up to her ear, with the same wrist that was wearing her own, she’d heard a ticking sound. She looked at the side of the brass watch to see if she could wind it up.

“Well, that’s curious. There’s no knob to wind it up with.”

“I’ve seen that sort of instrument before,” said Brother Charles. “It’s not a pocket watch that works by springs. I think it’s a timepiece that works by the sun.”



“Oh, you mean it’s a solar-powered watch. But it looks so old-fashioned. And besides you don’t have solar powered things in your world. They haven’t been invented yet.”

“What I mean is that it’s a portable sundial – powered by the sun. Open it up.”

Alison opened it up and said, with disappointment, “oh it’s not a watch after all. There’s no dial and no hands. It’s

just a lot of rings within rings. But it is beautiful.” Indeed it was. It was made of brass, with several small diamonds in strategic places, and a small lens on one of the rings. “But it doesn’t look like a sundial either. Where’s the bit that sticks up?”

“You mean the gnomon,” said Emily, showing off as usual. “Yes, where’s the gnomon that casts the shadow?”

“There is no shadow. The light comes through that little lens and lands on one of the rings,” explained Brother Charles. “I saw something like that some years ago.”

“Well, what I do know about sundials is that they have to be placed in exactly the right direction – facing due north, or is it due south. How can you have a portable sundial, unless it comes with a compass?”

“No need for a compass,” explained Brother Charles. He seemed to know so many things. “You point it towards the sun.”

“Wouldn’t that blind you,” asked Alison.

“Ah, but you don’t look through the lens. The sun shines through the lens and is focussed into a bright spot. You see, the sun doesn’t rise in the east and set in the west – not exactly. And it doesn’t move directly overhead.”

“Unless you’re at the equator. But otherwise it travels around the north.”

“But, my dear sister, have you forgotten that we’re in England now?” Alison was eager to point out when Emily had made a small mistake. “In the northern hemisphere the sun travels from east to west, but via the *south*. That’s why people here like houses that face south, while in Australia they like north-facing houses. That way they get more sun.”

“Exactly. And the further you go from the equator the lower in the sky the sun travels. Also in summer it goes higher than in winter. So, if you know your latitude and the time of the year you can predict the path that the sun will take.”

“I see,” said Emily and if you see how high the sun is above the horizon you can tell what time it is.”

“Precisely. This instrument does all that for you. You turn this dial to the month of the year and point it towards the sun. The point of light that comes through the lens will shine on the time on this dial.”

“But you’ve got to have it very level.”

“That’s why there’s a spirit level there. You have to tilt it so that the bubble is exactly inside that little circle.”

Alison saw a small problem. “Won’t the height of the sun be the same in the afternoon as in the morning?”

Brother Charles laughed, “well, you’re supposed to know whether it’s morning or afternoon – as well as the time of the year.”

“But where do you dial up the latitude?”

“Oh, each one of these is made for a specific location. This one is probably made for London, which is close enough. I suppose someone really clever could incorporate another ring to set the latitude.”

“But what a business. It’s so much easier to have a watch with hands that go round.”

“Or a digital watch – or you can get the time from your smart phone.”

Brother Charles looked at Emily, having no idea what she was talking about. But he kept quiet and didn’t ask any

questions. He knew a lot more than they did about most things but from time to time they said things that completely baffled him.

Emily jumped up, holding the portable sundial. "Let's go outside and try it out." When Alison and Brother Charles both looked at her, with a certain look on their faces, she realised her mistake. And so the rain kept pouring down.

After a while Galois came in all wet and stood in a pool of water that was growing bigger as more and more water drained off him. "I think this would be a good place to live," he said. "I've just tested all the trees around here. They're just the right sort for building dams."

"I suppose this timepiece qualifies as my next axiom," said Alison. She took it as a foregone conclusion that all pieces of jewellery that they might find would be hers. Emily was no longer jealous. If the truth be known she'd have much preferred a good book to a piece of jewellery.

Brother Charles thought for a moment. "I suppose you could say that this represents the Power Set Axiom. That says you can have sets of sets of sets etc. All the sets within a set make up a set themselves. It's a bit like the boxes within boxes."

"And this timepiece runs on solar *power*, in a sort of fashion," added Alison.

Just then they heard a noise downstairs. Alison went to the top of the stairs and looked over. There were two rather ugly looking men.

A very rough-looking woman came into view, carrying what looked to be a very heavy bag. “A nice lot of loot,” said the woman. “Joe, you put it upstairs with all the rest.”

Alison dived back into the room where the others were and told them that they had to hide. Fortunately, in the room where they were there was a wardrobe, as well as the chest of drawers. It was certainly big enough for all of them to hide in, but the only problem was that there were lots of fur coats hanging up, which made it a bit uncomfortable.

“Let’s hope they don’t look in here,” said Alison.

“Ow! Don’t tread on my paws!”

“Sorry,” said Emily as she made her way through the coats. She pushed on the back of the wardrobe wondering if it led to a place like Narnia. Unfortunately it was solid and led nowhere.

“Shh!” whispered Brother Charles. They could hear the man, called Joe, emptying his bag into the drawers of the chest of drawers.

“Lucky we weren’t hiding there,” whispered Galois.

“Don’t be silly, even *you* couldn’t fit in those drawers. We’re safe so long as we keep quiet.”

“Someun’s been here – there’s all these empty boxes all over the floor.” After a little while they heard the footsteps recede. Alison peeped out – the coast was clear. But how were they to get out of the house without being discovered?

Galois had an idea. These big houses always had servant stairs at the back. If only we could find our way to the servants’ quarters. Galois led the way and they soon found a narrow set of stairs that led down to the kitchen. The outside door was locked, but fortunately it could be unlocked from

inside, so they escaped out into the rain. However the thieves had not gone, and our friends ran straight into their arms.

“What ’ave we here? A man, two girls, and what looks like a weasel. Lock ’em up.” They were taken to a shed in the garden and the door was securely locked behind them.

LOCKED IN

“Well, our goose is cooked this time,” said Alison gloomily.

“Don’t talk of food,” complained Emily. “I’m ever so hungry. I haven’t eaten since breakfast.”

“You’re beginning to sound like Ivy. Remember, it’s only lunch time,” Alison pointed out. “You won’t be starving yet?”

“Try telling that to my stomach. But, it might help if we pretend to be having a feast. Now talking about cooked goose reminds me of a recipe that Ivy told me about.”

The shed had probably once been a tool-shed, but no tools remained, which was a pity because they could have used these to break out. Moreover the thieves had taken Galois’ sword which might have been useful.

The shed was only made of timber, but it was very solidly constructed. There was a small skylight in the roof, but even if they could get up there and smash the glass the opening would have been too small, even for Galois to escape.



“What if they leave us here to starve?”

“Well there are no bones, so if they’ve used this shed before to lock people up, they’ve let them out before they starved to death.”

“Either that, or they’re very house-proud and clean out the bones each time ready for their next victims.”

“I just remembered,” said Emily. “I’ve got some mints in my pocket.”

“Not for me,” said Alison.

“Beavers don’t eat mints,” added Galois.

“Our only hope for escape is to work out how this lock works,” said Alison.

The lock was a variation on a combination lock. There were five brass tiles, each about an inch square. These slid around a three by two frame, with the sixth spot empty, revealing a small peep-hole. On each tile there was engraved a letter. They could slide these tiles around in the frame, and they could see that behind each tile there was a similar peep-hole.

“Can you see anything through the hole?” Emily asked Galois, who was standing on a box and peering out.

“The men seem to have gone. There’s a lane, but it’s a long way off.”

“Perhaps we could call out if we see someone pass,” Brother Charles suggested.

“No, we’re too far away to be heard.”

Emily examined the tiles. “I bet if we managed to put the tiles in the right order the door would unlock. We just need to guess a password. Alison, it’s like that puzzle that Dad gave us last Christmas – you know, it had fifteen tiles that slid around a four by four frame. Remember that there

were five patterns engraved on the back that you had to get, and the sixth one was marked IMPOSSIBLE.”

“But I did manage that one too.” said Alison.

“Only because you prised the tiles out of the frame and put them back in the required order!”

At that moment the tiles were as follows:

	N	K
L	O	U

“Well there are only 120 different permutations of those letters,” suggested Alison. “Couldn’t we just try them all?”

Alison knew this, not because she was good at mathematics, which she wasn’t particularly, but because she was a bell-ringer. When they rang a full peal of Plain Bob Doubles on eight bells there were 7020 different changes, as every bell-ringer knows. The heaviest bell always rings last in each change and the other seven go through all the 7020 possible permutations. This takes about 3 hours. Occasionally they would use just six bells. With the tenor ringing at the end of each sequence the other five bells go through all 120 arrangements.

“That’s all very well, but we can’t go through them in a systematic way,” replied Emily. We might miss some. No we have to guess the password. Now if we assume that they’ve picked a word we just have to rearrange these letters into that word.”

writing down lots of symbols in the little notebook that Emily always carried with her. At last Emily cried out, “we’ve done it. Let’s try it.”

They slid the tiles around, consulting the notebook from time to time. At last the words read

U	N	
L	O	K

“It’s a good thing it only needed an even permutation. Otherwise it would have been impossible. Only half of the 120 arrangements can be done.”

They tried opening the door, but it was still firmly locked. “Damn!,” said Emily. “So that wasn’t the password after all.”

But Alison had an idea. “Mostly they need to unlock the door from the outside. It makes sense that they would have chosen the password to be read from the outside. So if it is UNLOCK from the outside ...”

“Oh, sis, you amaze me sometimes. Why didn’t I think of that? Inside the password must be ...”

“KOLNU,” said Alison.

“Silly sis, they’re not all in a line. Think about it.”

“Oh, I see, it would be NUKOL.”

To help you see this I’ve drawn what the letters must be on the outside to read UNLOCK on the inside. If you’re still confused hold the book up to a mirror.

N	U	
K	O	L

Emily and Galois got their heads together, and soon they had the puzzle solved and at last the door could be opened and they ran out into the sunshine.

But the thieves must have heard them, because the next thing they knew they were being pursued across the fields. Galois couldn't run as fast as the others so he hid in a ditch. Luckily he wasn't very big so he was well hidden. The thieves ran past him, after the others.

They caught up with the others and tied each of them to a small tree. "Now where's that dwarf?" said Joe to the others. "What's that?" There was a peculiar sound coming from one of the trees.

"Lean backwards with all your might," Galois called out to Brother Charles as he ran away from the trees. Charles leaned back and there was a sharp crack. The next second he found himself being pulled over backwards as the tree fell over. Galois had gnawed through the base of the tree and it didn't take much for the tree to fall.

As the tree fell, Galois nimbly jumped out the way. But Joe's lady friend was directly underneath as it came down. Luckily for her it was a small tree – so the blow wasn't enough to kill her. But it knocked her to the ground and she had a headache for days. The two men went to help her,

which gave Galois time to come back and gnaw through the ropes.

Fortunately the two men were too busy pulling the woman out from under the tree and attending to her that they didn't seem to notice that Brother Charles had escaped. He limped to where the other two were still tied and, with his penknife, cut their ropes.

"Hey, they're getting away!"

"Leave them. If they go to the police we'll be miles away. They won't go back to the house and by the time the police arrive we'll have taken the stuff to another hide-out. Who was it who left the door unlocked?"

After a great deal of walking, our friends came to New Romney where they found the police station. Here they gave their report of what happened, and gave directions as to where the police could find the derelict house and all the stolen goods.

"Jump into that police car," said the sergeant, "and show us where to go. Then we'll take you back home. I can see that you've somehow got a few bruises."

The sergeant and his constable got in the front, while the four escapees squashed into the back. Luckily Galois was small, so he sat on Alison's lap. Now they couldn't go the way they'd walked, across the fields, but Galois had a good sense of direction and was able to show the sergeant where to drive.

But, of course, there was no sign of any stolen property when they arrived. "Are you sure you weren't just imagining all this?"

Alison remembered that she had inadvertently put the portable sundial in her pocket. She gave this to the constable, who examined it closely.

“I say, Bob,” he said to the sergeant. “This looks like one of the items the major reported as having been stolen the other night. It seems they *were* here. They must have cleared everything out”.

The sergeant took the portable sundial and gave Alison a receipt – and so ended another day of adventure.

The next day the sergeant of police came to where they were staying in Hythe. He told them that the three thieves had been found. He said that, hearing the story of the tree falling, he had the idea of going to the nearest hospital. And, sure enough, there were the two men explaining to the doctor how the woman had fallen down the stairs and hit her head. The stolen property was recovered.

They left that evening for their long journey back to St Huberts. It took two days, and when they got back, a small packet was waiting for them, addressed to Alison. It was from the Kent police. They had returned the stolen items to Major Campbell. When he heard of the portable sundial he said, “Och, give it back to the wee lassie. I couldna work out how to make that damn thing work.” So the latest of Alison’s axioms was now in her possession.

VENTRILLO CAT

The search continued for Ramanujan's ring. Now, the intelligence network in the Catholic Church is very efficient, and it came to the ears of Brother Charles that the thieves had hidden the ring, but had left a clue for their confederates in a book called *Nilpotent Rings*. You see the word 'potent' means 'powerful' and indeed it did seem that Ramanujan's ring was very powerful. But they thought that one of the thieves might have hidden a clue for another of their gang by putting it in such a book. Clever detectives might have expected a clue to be hidden in a book called *Potent Rings*, but *Nilpotent Ring* would suggest a ring that wasn't at all powerful.

Another piece of intelligence that came to Brother Charles ears was the fact there was a copy of this book in a circulating library in Upper Middleton – a village not so very far away.

That's why God gives us two ears. We hear something in one ear and something else in the other. Our brain, in between, puts two and two together and gets four. That is unless one has studied higher arithmetic, when under certain circumstances two and two can be something quite different!

"Upper Middleton – sounds like a contradiction," Emily argued. If it's 'upper' it can't be 'middle' and vice versa."

"What about the phrase 'upper middle class'?" asked Alison. Why I heard you tell somebody not so long ago that

our family was upper middle-class.” Emily went quiet, a sign that she accepted temporary defeat..

So one bright morning she and her sister set out with Ivy Dunn and Isaac, a packed lunch and a large helping of optimism. They were walking through a forest when Emily noticed a pair of eyes sitting just above one of the branches, staring at them.

“I think we’re being watched,” she said. At this a face appeared around the eyes, and then a body appeared underneath the face. It was a cat.

“Rats,” said the cat, “I must learn to close my eyes.”

“But then you wouldn’t be able to spy on us.”

“True, but never mind – my name is Isaac,” said the cat, “and yours?”

“I’m also Isaac,” said young Isaac. “What a coincidence.”

“A coincidence is where two things happen together and where there is a very small probability of that happening,” explained Isaac the cat.

“No I meant what a coincidence.” Anyway they introduced themselves. Then Alison asked him, “what sort of people live about here?”

“In *that* direction,” the cat said, waving its right paw round, “lives the Pieman, and in *that* direction,” waving the



other paw, “lives an Echidna”. Visit either you like: they’re both irrational.”

“But I don’t want to go among mad people,” Alison remarked.

“Oh, you can’t help that,” said the cat: “most of us here are irrational. I’m irrational. You’re irrational.”

“How do you know *I’m* irrational?” said Alison.

“Do you have a numerator and a denominator?” asked Isaac.

“I don’t think so,” said Alison.

“Then you can’t be rational,” said Isaac.

“I get it,” said Emily. “A rational number is a fraction, like $\frac{3}{4}$. It has a top number, called the numerator ...”

“But what has that to do with *people*? And how do you know that *you’re* mad?”

“I didn’t say ‘mad’,” said the cat. “I said irrational”. I’m the square root of two and if you have ten minutes to spare I can prove that the square root of two is irrational.”

“We can’t await for all that nonsense,” said Ivy. “I ‘ave to get sumpin inside o’ me afore I faint. I say we go find the Pieman.”

“Then I must bid you farewell,” said the square root of 2 and both he and his eyes disappeared. Alison waited a little, half expecting to him reappear, but he didn’t, and after a few seconds Emily said, “come on, we can’t wait here all day. Which way shall we go?”

“Well, as I said, the Pieman sounds promisin’,” said Ivy, “I aint had nuffin since brefast and I’m fair famished.” So that’s the way they went.

They had not gone much farther before they came in sight of the house of the Pieman. Alison thought it must be the right house, because it was like a gingerbread house, though the walls were built with pies instead of bricks.

There was a table set out under a tree in front of the house. It was in the shape of a cross, like the Red Cross, but only white – there was a white tablecloth. It had horizontal and vertical axes.

The Pieman, at least Alison presumed that he was the Pieman because he was wearing a chef's hat, and a giant Echidna, were along one axis. I don't know if you've ever seen an echidna in the Australian bush, but I warrant you've never seen one over six feet tall! Off to one side, on the other axis, was a pair of eyes, floating above one of the chairs.

"I've seen those eyes before," said Emily, as she looked at the floating eyes. As she said this a face appeared around the eyes and it was the cat. "I thought my disguise was impenetrable," said the square root of 2.

"Let me introduce you to my real friends. This is the Pieman and this is the Echidna."

"And you are the square root of two."

"Actually I lied. I'm really the square root of Minus One. He's my father."

"But there's no such thing," said Alison.

"Well they call me imaginary – but in mathematics even imaginary things exist."

By this stage the Pieman had got up and was shooin' them away, calling out "there's no room for you here. We are complete."

“Stuff an’ nonsense,” said Ivy indignantly, sitting down in the nearest chair. This happened to be next to the Echidna.

“This party is by invitation only,” explained the Echidna. “Only irrationals are invited. Why we didn’t even invite our good friend Minus One.”

“That’s because he’s not irrational,” explained the Pieman. “He’s a bit negative but not exactly irrational.”

“But these, my new friends here, are *all* irrational. None of them has a numerator, and none has a denominator.”

“I’m not sure what them things is,” said Ivy with her mouth half full. “But I’m sure if I *did* know I never did ’ave ’em. I ’ad a deprived childhood you know and I never ’ad lots o’ things.”

“Well there you are,” declared the cat to his friends and so, with his permission, Alison and Emily sat down.

“Let me introduce them,” said Isaac, “as you know *i* is irrational.”

“I think you mean “I *am* irrational.”

“No, *i* is my nickname, short for Isaac. You obviously didn’t notice that the ‘*i*’ was in lower case. Well, I (upper case) am *i* (lower case) and we are irrational.” Then he pointed to the Echidna. “And *e* is also irrational. I mean *e*, not ’e, short for ‘he’.”

“What about the Pieman?” asked Ivy, reaching for another pie. “Is ’e mad too?”

“*He* is π and π is irrational too. Would you like a slice of pie?”

This invitation was unnecessary for Ivy, who was already onto her third. Alison looked at what their three

strange hosts were eating. Their pies had been cut open and she could see what they contained. The cat's pie was stuffed full of mice – whole mice that is, with their tails still attached. The Pieman's pie was full of even smaller pies, and looking closely at one of these that was open, she saw that it was full of even smaller ones.

“Like my set of Russian dolls,” said Emily. “I bet it's pastry all the way through.”

The Echidna's pie was full of ants. I suppose this was natural because an Echidna is also known as a spiny ant-eater. “It makes a change from Australian picnics where the ants invade from the outside,” thought Alison.

None of these pies looked very appetising. They couldn't see what Ivy was eating. Either she had something less disgusting, or perhaps she didn't mind *what* she ate! By this time the Pieman and the Echidna had gone into what looked like a trance. Isaac explained that they were meditating. “They're transcendental, you know. Now I suppose you've come here for a purpose – besides sharing our pies.”

“We're looking for a ring,” said Emily. At this the Pieman, and the Echidna, woke out of their trance. The Pieman reached out across the table and seized a bell – the sort a Pieman uses when he's walking the streets with his tray of pies calling out “Fresh 'ot pies for sale!” This he rang vigorously.

“Not that sort of ring,” protested Alison. “The sort you wear on your finger.” The Pieman then tried to balance the bell on his big finger.

“A *round* ring, a ring of gold with a ruby.”

“Not a nilpotent ring? Not a ring of bright water? Not a ring a ring o’ rosy?”

“No, a piece of jewellery ring – the Ring of Ramanujan.”

“I did hear about that once,” said the cat. “But I’m afraid you’ve been wasting your time. We have no idea where it is. But you could try Upper Middleton. It’s in that direction.”

They set off believing that they *had* wasted their time. That is, except for Ivy, who still had crumbs on her face.

After walking along a narrow lane they found a path that Emily was sure would take them to Upper Middleton. At



last they came upon an old man sitting on a stile. He reminded them of the crooked man of nursery rhyme fame. He was twisted with arthritis and indeed, he had a mangy, cat beside him that could well be described as ‘crooked’. All that was missing was the crooked sixpence.

“Where are you three ladies walking to?” he asked, in a crooked voice. “Or should I say ‘to where are you three ladies walking?’ Are you just walking there and back just to see how far it is?”

Now, the curious thing about this man's voice wasn’t so much its crookedness, but the fact that it didn’t quite seem to match the movements of his crooked mouth. It was like a film where the picture and sound are ‘out of synch’.

“We’re walking to Upper Middleton,” said Emily. “Pray tell us are we on the right path?” Alison had never heard her sister ask with a ‘pray tell’ but somehow the quaint little antiquated man seemed to demand quaint, antiquated language.

“That depends on *why* you might want to go there?”

“But how can that possibly be?”

“If you’re going there to visit the monastery, you have to go back the way you came, because there’s no monastery in Upper Middleton.”

“Well we’ve just come from the monastery, and we need to go to Upper Middleton to visit the circulating library.”

“In that case you need to go over this stile and follow this narrow windy path for about a mile. But before I can let you pass I have a tale to tell you – one with a twisted ending.”

Emily and Alison were in no hurry, and besides they loved stories, especially those with surprise endings. So they sat down on the grass and asked him to begin, which he did in his strange, out of synch, voice. Meanwhile the cat just rolled over and began to groom itself.

“One fine day in the middle of the night – stop me if you’ve heard this before.”

“I think I have,” said Alison politely.

“Rubbish, you can’t have. I’m just making it up for the first time. Let me go on. One fine day in the middle of the night”

Emily objected: “But how can *that* be? Was it *day* or *night*?”

“Both,” he explained.

“But day and night are mutually exclusive events,” she protested. “It can’t be *both* at the same time.”

“Well it was ten o’clock in the morning by the watch on my left arm and ten o’clock at night by the watch on my right. You see around here we use two different times simultaneously. We look at both watches and choose whatever time we want it to be.”

“But surely both watches must have been telling the same time,” objected Emily. “Ten o’clock is the same whether it’s AM or PM – big hand on 12 and ...”

“If you keep interrupting me I’ll never finish my story. One fine day in the middle of the night ... I met a man in tears and she told me ...”

“Wait there,” said Alison. “Was it a *man* or a *woman*?”

“Yes,” replied the old man.

“Was it a man?”

“Yes, didn’t I say so?”

“But then you said ‘she’. Was it a woman?”

“Of course you dummy – ‘*she*’ usually means a woman.”

“So she was *both*?”

“Yes, of course,” replied the crooked man impatiently. “Aren’t you two going to let me finish the story?”

“But I just want it to be logical, or else I can’t understand it. You mean it was one of those hermaph ... one of those persons of uncertain gender.”

“No, not at all. She was definitely a man.”

“But that’s like saying that something can be both true and false at the same time!” Alison was starting to become agitated.

“And what’s wrong with that?”

“Where *we* come from a statement can be true or false, but never *both*.”

“I see you’ve never heard of quantum logic?”

Emily’s mouth dropped open. Here was a rough country bumpkin who had heard of the word ‘quantum’. She had only heard of it recently from her father, and *he* was a mathematician.

“Is light a series of particles or a wave?” the old man asked.

This was deep stuff. Alison had heard father talk about how in some situations light was a stream of photons. In other cases it acts like a wave. But surely a statement that is both true and false at the same time is impossible. It went against all she believed about truth. “You can’t have a contradiction like that.”

“I remember father telling me a story once,” said her sister. He said that if you assume that 2 plus 2 is 5 then he could prove that I was the Pope. How does it go again Emily?”

“Suppose that 2 plus 2 is 5. But we all know that 2 plus 2 is 4. Therefore $4 = 5$. Subtract 3 from both sides and you get $1 = 2$. The Pope and you are *two* persons. Therefore the Pope and you are *one* person. Hence you are the Pope!”

“And indeed I am. Alex Pope is my name.”

“But how can two persons be the same person?”

“Easy. A certain person in my family is Alex Pope. *I* am Alex Pope. Does that prove that I am my sister?”

“I see ... your sister’s name is Alexandra and you are ...”

“... tired of all these interruptions. I’ve lost my train of thought. Never mind I’ll wait till the next one comes along. Let me see. One fine day in the middle of the night I met a man in tears and she told me that her two dead brothers were fighting. Back to back they faced each other. They drew their swords and shot each other. A blind man went to see fair play. A dumb man went to shout ‘hooray’.”

Emily laughed. “Oh Alison, can’t you see? It’s just that nonsense rhyme our cousin told us once. It’s not *supposed* to make sense. This old man is just making fun of us. Let’s go.”

“Perhaps I am and perhaps I’m not,” he said. But you can’t prove that I *am* and you can’t prove that I’m *not*. But I *can* prove that you’ll never be able to prove it one way or the other. Even *I* can’t prove whether or not I’m pulling your leg. It’s what’s called in mathematics an undecidable statement. You’re logically free to accept it or deny it.”

“We must ask you to allow us to pass,” demanded Emily, getting a little cross. The old man stood up on his crooked legs muttering something under his breath and walked away.

“Cranky old man,” said the cat. Alison and Emily stared in amazement. Not just the fact that a cat could talk, not just because its voice was that of the crooked man who was by now a long way off, but because it was Isaac, the vanishing cat they had met earlier on.

“So it was *you* doing the talking all this time. That’s why the old man’s lips were out of synch.”

“Indeed it was I,” replied the cat in a rather less crooked voice. I’m what you call a ‘ventrillo cat’. Did you notice that my lips didn’t move?”

“I can’t say that I was looking,” said Emily, still stunned with amazement.

“That old man is dumb. He hasn’t spoken for thirty years or more.”

“But he said he was Alex Pope, and I thought you were Isaac.”

“He is Alex. I was only his voice.”

“Well you didn’t finish the story. The old man – I mean you – promised us a twist at the end of the tale.”

At this the face of the cat started to disappear, and then the body, until all that was left was just a disembodied pair of eyes, and a tail. Then the eyes vanished, and all they could see was the crooked tail, moving back and forth like a metronome.

“I get it,” said Alison. “You don’t come from Cheshire by any chance?”

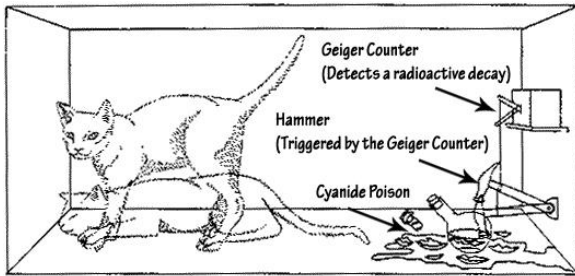
“No, no,” said the cat as he began to reassemble himself. “I’m a Swiss cat – from Geneva. I just love Swiss cheese!”

“I’m amazed,” gasped Emily. “Not that a cat can talk, but that a Swiss cat could talk English with no accent.”

“Je suis un eggcellent meemic,” he said in a French accent.

“So what brought you to this country?”

“My master, back in Geneva, was very cruel to me. He kept putting me in a box with a bottle of poison and a lump of something. He was heartless. From inside the box I would hear him say to his friends ‘if even one atom of that radioactive substance decays then the mechanism will release the poison and the cat will be dead. But because radioactive decay is unpredictable we can never tell whether the cat is alive or dead – he is therefore simultaneously both alive and dead’.” Well, I knew that I was still alive so it was no mystery to me!”



“I thought that it was only a *thought* experiment.”

“Really? I assure you that it seemed real to me!”

“And so you survived.”

“Often I survived, but sometimes I died. Of course once the box was opened I was no longer in a superposed state. I was either completely dead or completely alive.”

“But how could you come alive after you were dead. That’s impossible!”

“So you don’t believe in resurrection?”

“Well, yes, but the resurrection of Jesus was special. Cats don’t resurrect.”

“Haven’t you heard of cats having nine lives? What do you think that means? Anyway it’s no great feat to resurrect a cat. To resurrect a *person*, that’s more of a miracle. But when Jesus was in the tomb? Was he dead or alive?”

“Well he was dead when he went in and alive when he came out.”

“So for the three days he was in the tomb he was simultaneously dead and alive,” said the ventrillo cat, “just like me in the box.”

“How do you mean?”

“Well, he was dead – after all he succumbed to crucifixion. But we all believe in the Trinity – three in one. So he was one with God. Now if God was dead for those three days the universe would have imploded. It’s only the energy of God that keeps it going, you see.”

“My you are a theological cat! But tell me how you came to have such a crooked tail.”

“Well, when I ran away from my master – you see I was onto the last of my nine lives so I couldn’t chance Dr Schrodinger’s box again – I ran away to sea. I survived on the ship’s rats. But one day I was too close to the anchor as it was being raised.”

“I nearly became a Manx cat but a kind sailor bound up my tail with a rag and some pitch. My tail healed but because he wasn’t a skilled surgeon it grew crooked. Never mind, I’m grateful to him for having saved my tail. All in all I served as a ship’s cat for several months. But one day, in Liverpool, I jumped ship and decided to try my luck on land.”

“Well England does offer a lot of opportunities, even for a cat.”

“Oh yes, I heard that Dick Whittington’s cat became Lord Mayor of London.”

“I think you’ll find that it was Dick Whittington *himself* who became Lord Mayor,” corrected Emily – a stickler for accuracy, as usual.

“Well, I wouldn’t mind being just the Lord Mayor’s cat. Besides, I haven’t really got a head for politics. Now where did you say we were going?”

“We – that is Alison, Ivy and I, were trying to get to Upper Middleton. I don’t know to where *you* were going.” Alison winced as Emily bent over backwards trying not to end her sentence with a preposition.

“Where you’re going ladies, *I* will go. Your goal shall be my goal. I’m at your service.” At this he bent his crooked tail in front of him and gave a wonderful impression of taking a bow! “This way.”

It wasn’t a twisted mile, as they had been led to believe. It was a reasonably straight path, and it was certainly much more than a mile. After three miles Emily said “I thought you said it was only one mile.”

“Oh, no, that was a joke. I remembered the rhyme about the crooked man and the crooked mile. You see there are three types of statements – truths, lies and jokes. A joke, by definition, is a funny lie. No, I tell a lie. There are four types of statements – truths, lies, jokes and myths.”

“But doesn’t a myth qualify as a lie, that is, a fiction?”

“Oh, good heavens no. There are *true* myths and *false* myths. A myth is something that unlocks a deep truth. It doesn’t matter whether it’s literally true or not. The Greek myths were probably not literally true, but they reveal powerful truths when you look beneath the surface. The

Resurrection in the Bible is also a myth. I happen to believe that it was literally true, but with a myth there are deeper truths than their mere historical veracity.”

By now they had started walking along a dusty lane. After some time they saw a horse looking over a fence. “Good morning,” it said in a very proper English voice. “Wonderful weather we’re having for this time of year.”

Alison and Emily looked around in amazement. Then Alison laughed. “I suppose it’s Isaac having some fun.” Indeed it was. Whenever they passed any animal it greeted them in a different voice. But all the time it was Isaac.

“So your name is Isaac. Are you Jewish?”

“Well, that’s the name my friends back there know me by. But my real name is ‘Max’. Remember that I escaped becoming ‘Manx’ by the skin of my tail. At least that’s my English name. In Geneva I was known as Maximillian Wolfgang Frankenstein. I can trace my lineage back to the cat that Dr Frankenstein practiced on before he created his famous creature. You’ve heard of a cat of nine tails. That was named after my ancestor who had to be fitted with nine successive tails before Dr Frankenstein was satisfied. Sorry, that qualifies as a joke.

“And I suppose you have a sister called Min,” said Emily, continuing the banter.

“Oh, you’ve met Minerva, have you? Oh, I see – that was a joke.”

After some time they passed a signpost that said ‘Upper Middleton 1 mile’. “I see you don’t have much

respect for logic if you believe that something can be both true and false at the same time – or were you just putting words in the old man’s mouth?”

“Oh, no I have a healthy respect for logic. The world couldn’t exist without it. It’s just that logic doesn’t explain *everything*. It has more the consistency of treacle than of steel. There are worlds far beyond our rational universe. I like the idea that there’s mystery beyond mere logic.”

“That’s a pretty deep thought for a cat!”

“Don’t forget why the Egyptians worshipped us. We were having deep thoughts while the Egyptians were running around building pyramids. Even today it’s assumed that when we’re lying down in front of the fire, with our eyes closed, we’re just asleep. Not at all! We’re deep in thought about metaphysics, or existentialism. Why, we have our best thoughts when our eyes are closed.”

By now they were on the outskirts of a village. There was a pond with little fluffy ducklings swimming among the reeds. These all said “good morning” in high pitched voices – all courtesy of Max of course.

At last they came to the outskirts of the village. “This is Upper Middleton,” announced Max, resuming his normal voice. “But I have no idea where you might find your circulating library. Maybe it’s in a windmill – the sort that can rotate so that it always faces the wind. You’d better ask somebody. I have to go and chase the mice from under my mistress’s chair.”

At this he began to disappear. First his face and then his body, until all they could see were his eyes and the crooked tail. The tail waved goodbye and vanished and the eyes winked. Finally they too vanished and they could neither see, nor hear, anything.

The village was quite small and didn't at all look like the sort of place that would have a circulating library. Neither of the two shops had heard of such a thing.

"We don't read much round here," said the lady in the tiny post office which also served as a general store. "We don't write much either. The last stamp I sold was two months ago."

"Wait there," said an old lady who was in the shop buying some wool. "They *do* read over at Middleton Manor – but they have their own library. Ever so big it is."

"Well they might know where there's a circulating library, even if they never need to use it," suggested Emily. "Where is the manor?"

The old lady screwed up her face as if this was the most difficult question in the world. "I did happen to walk past it once when I was a girl. Now where was it?"

"It might be on that hill over beyond the church," suggested the shop lady. "I remember mother calling it Middleton Heights."

"Oh, yes, that might be it. Just go down the road here until you come to the church. Turn right, and you'll see a hill in the distance. But mind. It's ever so steep, and ever so crooked. You might find Middleton Hall up there."

The two girls, and Ivy, thanked the ladies and set off. “Upper *Muddleton* would be a better name for that little village. They didn’t seem to know whether it was Middleton Manor, or Hall or Heights,” said Emily.

“And they don’t seem to travel much. It can’t be that far,” commented Alison. “Fancy them not having been there in years.”

They turned right at the church and found a sign reading ‘Middleton Hall 1 mile’. The road went up a gentle slope – not what you’d call a hill. It wasn’t quite straight, but not what you’d call ‘crooked’.

In a few minutes they found a large, old, house, with a tall round tower at one end. There was a high wall, but the gate stood open and they were able to walk up to the large oak front door. They pulled the bell and suddenly they heard some bells, as if this was a large church and there was a wedding.

“I do believe that’s Steadman Triples on eight bells” said Alison, who had done some bell-ringing back at All Saints in Lancaster Gate. They must have a band of eight ringers up there in the tower.”

The sound was so loud that the good people of Upper Middleton must surely have heard it. So why the location of the house was a mystery to the residents in the village was itself a mystery to the two girls.

They waited for several minutes and eventually the ringing ceased. They were relieved that they didn’t have to wait for a quarter peal, which would have gone on for forty-five minutes. A sound of one metal surface scraping against another could be heard, as if the door hadn’t been unlocked in

years. Finally the door creaked open and the old crooked man that they had seen before, appeared, carrying a lantern and a pad of paper. They remembered that he was dumb.

‘WHAT MIGHT YE BE WANTING LADIES?’ he wrote on a pad.

“We’re looking for the circulating library, Mister Pope?”

The crooked man reached for his pad and laboriously wrote:

‘I AM NOT THE POPE – MY NAME IS MEANEY
SINGER’

“But you said that you were Alexander Pope when we met you earlier,” protested Alison.

‘IT MUST BE THAT PESK –K’ Here his pencil lead broke off and he had to reach into his pocket for another pencil. ‘Y CAT – NEVER BELIEVE ANYTHING HE SAYS.’

This conversation was getting so painful that Emily thought they should get to the point. ‘We’re looking for a circulating library around here. The old ladies in the village said you might know where it is.’

‘THAT WOULD BE MAISIE AND MILLIE’, wrote the old man. ‘I HAVEN’T SPOKEN TO THEM IN YEARS.’

“Well, is there such a thing as a circulating library round here?”

‘HA! HA! THAT’S A GOOD JOKE,’ wrote the crooked man, ‘CIRCULATING LIBRARY – *ROUND* HERE’.

The conversation was painfully slow, because the crooked man’s fingers were crooked with arthritis. Eventually

he said, I mean he wrote, that he'd call his brother. He pulled a bell pull and the bells rang again. At last a second crooked man shuffled into the room. "Perhaps I can help," he said.

"We're looking for a circulating library around – in the vicinity. Do you know of such a place?"

The second man looked towards his brother as if he had no idea what they were asking. His brother wrote the question on his pad, and added: 'HE'S DEAF YOU KNOW.' The second brother's face lit up as he read the question. "You've come to the right place," he said. We have a library in the round tower. Were you looking for anything in particular?"

"We've been told to look for a book called *Nilpotent Rings*," Emily replied, looking at the dumb brother. He wrote the question on his pad, tore it off and handed it to his deaf brother. That brother crumpled it up and discarded it on the floor, along with all the other conversations. "I expect that would be under J for jewellery ... if we have it, I'd better ask my brother."

But instead of turning to the other brother he rang the bell and a *third* brother appeared, shuffling into the room with a white cane.

"So there are three of you. Are you all brothers?"

The first brother wrote 'YES' on his pad. The second brother just stared at them, not having heard the question and the third brother said, looking in the opposite direction to where they were standing, "yes we are three brothers. I'm Eaney, and I'm blind. That's Meaney, and he's dumb, and that's Miney and he's deaf."

That explained a lot though, not being able to see his brothers he mixed Meaney up with Miney, so that they were all confused. Let me make it clear. Eeney was blind, Meaney was dumb and Miney was deaf, just like the three wise monkeys.

“Eeney, Meaney, Miney,” said Alison. “Where’s Mo? I suppose he’s lame.”

“Oh no, Montgomery couldn’t be a world class opera singer if he was lame. No he’s in London, singing at Covent Garden.”

“So there’s nothing wrong with him?”

“No, of course not. What makes you think he has some sort of disability?” asked the blind man. “Now I seem to remember that book. It’s under M for mathematics.”

Alison’s face lit up. “That sounds right. Can you find it for us?” Eeney said he could, but being blind he had to ask Meaney to help him find it. Meaney pointed to a door and they all followed him up a spiral staircase.

It soon became clear why this was called a circulating library. It wasn’t a library that lent books but one that was circular in shape or, more accurately, it was a helix. I had to make that clear before Emily showed off her extensive knowledge of geometry and told you herself. The walls of the spiral staircase were lined with bookshelves, both on the left and on the right.

Like all spiral staircases it turned to the right as you went up. This is a design that’s used in most fortified castles. It allows those defending the castle, mostly right-handed, the ability to use their swords freely in their right hands when descending to face the enemy.

Under the A's they found *Alice in Wonderland* and when they reached the M's they could only seem to find books like *Merlin and King Arthur*. Meaney realised their mistake and wrote on his pad 'THIS IS THE FICTION STAIRCASE' and showed it to Miney, who said, "we have to go back down – this is the fiction staircase."

They retraced their steps and when they reached the bottom Miney took them one hundred and eighty degrees around the base to where there was a second spiral staircase that intertwined with the first. Miney explained that this tower followed a design of Leonardo da Vinci's, who was obsessed with spirals. This time, under A, there were books on Astronomy and, when they reached the M's several floors above they found, at last, the Mathematics section.

"*Rings and Radicals, Non-Commutative Rings*," said Emily under her breath, as they scanned the shelves. "Here it is, *Nilpotent Rings*."

Eagerly Alison thumbed through the pages, looking for marginalia – things written in the margins. There were a few minor corrections, pointing out that a certain x should be a y .

In one place someone had written "there is a counterexample to this theorem" followed by a whole lot of symbols. Another entry was the statement "indeed this is true for all n and I have a wonderful proof of this but the margin is too small to contain it".

At last Emily squealed with excitement. There, at the bottom of a page, in a rather uneducated hand, were the words "Ramanujan's Ring is ...". Eagerly she turned the page, only to find "... is a very powerful thing, but if you

think I'm going to tell you where it is you're very much mistaken."

So it had all been a waste of time. Gloomily they said goodbye to their three hosts and went back to the monastery. They had plenty to discuss, such as what they might do next in their quest for Ramanujan's Ring. But there were other mysteries. There appeared to have been only three occupants of the house and yet they had heard Steadman Triples on eight bells whenever a bell pull was pulled. Was it a recording? Or perhaps it was some sort of mechanical device. And does logic have the consistency of treacle? Well perhaps not treacle, but more the consistency of glass which, as Emily was pleased to point out, appears to be a solid but has some of the qualities of a liquid.

THE INFINITY FISH

They had had a somewhat successful adventure, though they were no closer to finding Ramanujan's Ring. A few weeks went by and one day Brother Charles got an anonymous letter that suggested that the ring might have been thrown into the canal at Waltham Abbey.

Galois had a plan. "If we can persuade the lockkeeper to drain a section of the canal we might be able to find it."

They borrowed two horses and a carriage from the Monastery. The carriage was the one they kept for ceremonial purposes and for carrying special guests, such as Bishops and Cardinals. After a substantial breakfast they all set off – Alison, Emily, Charles and Galois and Ivy. Ivy was only coming with them as far as Tottenham because she had to take the carriage back in time for her to supervise the evening meal.

She was allowed to take the reins and they almost galloped to Tottenham – Ivy loved speed. I'm not sure whether the horses were so happy, but she promised them a good feed and a rest before the return trip.

At Tottenham they waved goodbye to Ivy and hired a narrow-boat, and another horse to tow it, and they set off northwards. Emily led the horse and Galois steered. At each lock Alison and Charles got off to open and close the lock gates.

They went through the Tottenham Lock and the Stonebridge Lock. There were no locks for some little time, so Emily went into the galley and made tea. After a while they came to the Picket Lock, the Ponders End Locks, the

Enfield Lock and the Ramney Marsh Lock. Just before the Waltham Town Lock they tied up. As it was getting on in the afternoon they decided to stay there overnight.

In the meantime they decided to walk along the towpath a little way and go fishing. The Ramanujan Ring would have to wait until the next day, because it meant talking to the lockkeeper, and he had finished for the day.

It was pleasant sitting on the boxes they had brought from the boat, while they took it in turns with the fishing rod. For about half an hour they had not the faintest nibble. Then, while it was Emily's turn to hold the rod, she felt it jerk in her hands.

"Quick, Emily, you've got a bite! Pull it in."

She pulled the rod quickly and a fish on the end of the line fell onto the grass.

"My goodness," said Charles, taking the fish off the hook. It's an *eternity* fish. I didn't know there were any in this country."

"I see what you mean," said Emily. "The pattern on its scales looks like lots of infinity symbols."

"It just looks like an 8 to me," said Alison.

Charles went on to explain. "Well they were known as eternity fish by the early Christians. You know how they adopted the symbol of the fish as their secret sign. You see, the Greek word for fish is ΙΧΘΥΣ and this was used as an acrostic for the phrase 'Ιἰῆsous Christos Theou Υἱος Σᾱτῆρ', meaning 'Jesus, Christ, Son of God, Saviour'. Also, omega is the last letter in the Greek alphabet and it was often associated with eternity. 'I am the Alpha and the Omega saith the Lord.'"

To make it easier for you to see the similarity between these symbols here they are:



small omega



infinity



Christian fish symbol

So when the early Christians saw the fish with this pattern they called it the ‘eternity fish’. It was only in the 17th century that the mathematician closed up the omega and it became the symbol for infinity.

“Well, Alison,” said her sister. “I suppose Brother Charles will tell us that this fish is an axiom – perhaps the Axiom of Eternity.”

“But a fish isn’t a piece of jewellery,” he replied. “Let’s open it up. You never know, it might have swallowed Ramanujan’s Ring. Wouldn’t be amazing if we opened it up and found the very ring we’re looking for?”

So Brother Charles cut the fish open and, would you believe, inside was something shiny and gold. No, it wasn’t the ring. It wasn’t any sort of ring. It was a very fine gold necklace. Each link was shaped like the infinity symbol.

“There you are,” he said. “The Axiom of Infinity.”

“It’s a pity there aren’t infinity many link in the chain,” said Alison. :With a bit of patience I could count them.”

“Yet if you start near the clasp and count the links till you get back to the clasp, how can you be sure that the next link is exactly the same one you started with? Remember

when we climbed that spiral staircase in Paradox Castle. We weren't sure whether we had got back to where we started."

"But that's silly," said Alison. "Of course it would be the same link."

"That depends on what you mean by 'same'. If you find two identical copies of a book in a bookshop you'd say that they're the 'same' book, wouldn't you?"

"Well, yes," said Alison.

"Yet you could buy one and there'd still be one in the shop. They can't possibly be the same book."

"But I'm not saying that the links are all the same because they're identical. The hundred and first link, assuming there are a hundred in the chain, would be absolutely no different from the first," protested Alison.

"Except, that by the time you got back to the first link it would be many seconds older. The difference between the first link you counted and the hundred and first would perhaps be a minute! You see, if you take time into consideration they *are* different."

Emily got the idea, a little faster than Alison. "Yes, after all *you* are not the same Alison who got onto the train in Leinster Gardens at the beginning of all this. You're a little older and, as you know the body keeps shedding cells and growing new ones."

"Yes, but a gold chain doesn't shed cells. It stays the same forever."

"You don't know that. The change may not be visible to the naked eye, or even under a microscope. No, if you keep counting forever, there will be infinitely many different links!"

“So I get to keep this as my next axiom, I suppose,” said Alison. “It’s the Axiom of Eternity”.

“Or rather, the Axiom of Infinity. There’s an axiom that essentially says that there’s a set with infinitely many things in it.”

They suddenly noticed that Galois had made no contribution to this discussion. The reason was that he had his mouth full and was eating something.

“Delicious.” he said.

“You haven’t eaten the eternity fish have you?” reprimanded Brother Charles.

“Of course not! Beavers don’t eat fish. No, these are some twigs I found just off the towpath. Your fish will be in that bucket I expect.”

“But I thought beavers love fish,” said Emily.

“Brother Charles will tell you why we don’t eat fish.”

“Will I?”

“Yes, didn’t you know that the Catholic Church has defined a beaver to be a fish?”

“I never heard of that,” said Charles somewhat puzzled. “What a bizarre thing to do. Are you sure you’re not just making that up?”

“I assure you it’s true, but you may not have heard of it because it was a decree made by the Canadian bishops.”

“Yes, but what business do bishops have with dabbling with taxidermy?”

“Well, beaver meat had been a favourite item of food for the North American Indians for a very long time – long before the missionaries came. When the European Catholics came and converted the native Indians they insisted that they

shouldn't eat beaver meat during Lent. The natives made such a fuss at this that the bishops decided to declare that a beaver was a fish. After all, they swim like a fish."

"But they don't *look* like fish and fish don't build dams," protested Alison.

"Anyway, we aren't fish. And even if we're only fish in name, according to the Church, it doesn't seem right for us to eat fish."

"But I'm sure beavers didn't change their eating habits just because of what a few bishops said. They must not have eaten fish for thousands of years."

"You may be right. I wouldn't know what fish tasted like. I prefer the outside layer of trees. Now *that's* real food!"

The next morning they consulted the lock keeper. He explained that in order to drain the section of canal between Waltham Lock and Waltham Common Lock he would have to write to the board of governors of the canal company and they would have to meet to discuss the proposal. It could take weeks.

So Charles asked him to go ahead and write. In the mean time they would have to return to St Huberts. So he sent a wire to the Monastery, saying that they would be back at Tottenham the next day, at about noon and asked that Ivy would be there to meet them with the carriage. Then they set off.

Now the trouble with narrow-boats, on a narrow canal, is that you can't simply do a U-turn. Usually the boat is longer than the width of the canal. This was the case for their

boat, *The Heron*. This meant that they had to continue going north until they could find a ‘winding hole’. This is a section of the canal that is made wider so that narrow-boats can turn round.

Brother Charles consulted his map. There was no winding hole for quite a distance. But he noticed that just above Feilde’s lock there was a junction, where the River Stort branches off. They could use this to do a ‘three point turn’.

So they continued north, encountering seven more locks. Then there was the difficult manoeuvre of doing the turn. Had the canal been a road, and the boat a vehicle, this would have been a matter of seconds. But when you have to uncouple the horse from the boat and do the turn by punting the boat with a long pole, the turn took over half an hour – especially as Brother Charles, who was doing the punting, fell in at one point!

Now they were heading south and, after going through the same seven locks again, they were back in Waltham Abbey. They decided to tie up here for the night. But, as it was still daylight, they went fishing. You never know, the next fish they caught might have happened to have swallowed the famous ring.

But, alas, all they caught were three fish. When these were cut open their stomachs revealed nothing more than what fish usually eat. Charles cooked them for tea while Galois found some bark and a few twigs.

The next morning they set off again. Back through the Ramney Marsh Lock, then the Enfield Lock and the Ponders

End Locks. This time Charles took the horse, Emily operated the locks, Alison was in the boat and, as usual, Galois just stood on the roof of the boat and directed operations. This time, at Picket's Lock they had a bit of excitement, which could have been disastrous!

You see, when you are going 'downhill' in a lock you first have to make sure that the water level in the lock is at the higher level, so that you can go in. If it's not – say if the last person to use the lock was also going downhill – you must close the gates at the lower end and then fill the lock up to your level.

This is what happened at this lock for our four friends. Charles untied the horse from the boat. Emily got off the boat and walked down to the lower gates, which were open. She then closed these gates. This involved putting all her weight on a long lever, connected to the gate. (It's a rather more energetic operation than just shutting a yard gate!) Then she had to get across to the other side of the canal, by a walkway on the lock gates, so that she could close the other gate.

Following this she had to go to the back gates, which were closed. Of course there is no way she could have opened them straight away because the pressure of the water would have made that impossible. She had to fill the bottom lock first, so that the level of the lock was the same as that of the canal where the boat was.

Now when you fill a bathtub you just turn on the tap. If this had been the arrangement it would have taken months to fill the lock. What she had to do was to open two large square holes that allowed the water to flow in from the upper level. These holes are covered by things called 'paddles', and these

have to be raised. The paddles are connected to a ratchet arrangement. Emily was holding a lock key – a large piece of metal, about one and a half feet long. This she connected to the ratchet and started winding, as if she was winding some giant clock.

This is hard work and Emily came out in a sweat. Once she opened the paddles, one on each side of the canal, water would start pouring into the lock. After about five minutes the lock was full and now it was easy to open the top gates. Well, I should perhaps have said ‘possible’ rather than ‘easy’. It was still hard work. Emily had to put her bottom behind the long lever and, digging her feet into the ground, push backwards with all her strength. Perhaps Brother Charles should have been doing all this hard work.

Once the top gates were opened Alison threw Emily a rope and she pulled the boat into the lock. Now she had to shut the top gates – more hard work – and go to the bottom gates to open the paddles there, to let the water out. The boat would then be lowered to the bottom level and Emily had to open the bottom gates and climb back on board. The horse could again be tied to the boat and they would be on their way again.

I expect you're thoroughly confused so it might be a good idea for you to use some blocks of wood as pretend lock gates and a pretend boat.

Just remember that this process has to be carried out at every single lock. It's no wonder that it takes a long time to go even a short distance, especially if the locks are close together. I haven't said anything about bridges. A normal bridge is one where the boat can go underneath. Usually the

horse can't go under the bridge and has to be untied and be led to just beyond the bridge. But on some canals there are opening bridges which involve rather more work. However it was fortunate that there were no opening bridges on this section of the canal.

Well, all went well until the top gates were closed and the lock started emptying. Emily and Alison must have both been daydreaming because Emily hadn't pulled the boat forward far enough. The back of the boat was almost touching the back gates and Alison, although she was at the back on the tiller, didn't appreciate the predicament they were in. Charles was looking after the horse and Galois – well what did he know about canals?

You see, at the bottom of the gates is a sill – a strip of concrete about eighteen inches wide. Now when the boat was lowered, the back of it rested on this sill. But the front was still going down, with the water. Alison noticed, to her

horror, that the boat started tipping forward.

“The boat's tipping over!” she cried out. Fortunately Charles heard the scream and called out to Emily as he ran towards her. “Pull the boat forward!” But it was too



late. The back of the boat was firmly resting on the sill and wouldn't budge.

Galois jumped off the boat into the lock and swam to the side, where there was a ladder built into the wall.

Charles had reached Emily by now and he snatched the key from her and ran back to the bottom paddles. Desperately he closed the paddles and the water stopped flowing out of the lock. By now the boat was tilting forward at thirty degrees and Alison could hear plates falling out of the cupboard and smashing onto the floor! She was holding onto the back of the boat so she wouldn't slide forward.

Charles then ran to the upper gates and opened the paddles. Water started pouring in and the lock began to fill. As he wiped the sweat from his brow he saw the front of the boat rising again until, at last it became level.

After a couple of minutes of getting over the shock they resumed the operation. Charles went back to where he had left the horse and found that it had wandered off. Fortunately it wasn't too far away, feeding on a juicy patch of grass. The boat was pulled forward, away from the sill, the lock was emptied, the bottom gates were opened and they were on their way. Alison and Emily had learned a lesson that day – keep clear of the lock gates – especially the back gate when you are descending.

“I heard of a case recently,” said Brother Charles where a boat sank at the Gargrave Lock on the Leeds and Liverpool canal, because the back had got caught on the sill. Three people drowned.”

The excitement over, they continued on their way. But Charles decided to swap with Emily – just in case.

OUTSMARTING THE GYPSIES

Once they got clear of Picket's Lock they tied up so that they could inspect the rudder. Galois was voted the most suitable for this job as he was already wet and, besides, he was the one most at home in water. He dived overboard and looked at the rudder. It was a bit bent, but he assured the others that it would do, at least until they got back to Tottenham. So he came aboard and changed into some dry clothes, and off they went.

As they set off Charles, who was still leading the horse, noticed that it was limping. He inspected the hooves and discovered that one of the horseshoes had come off. It would be very painful to continue. He had to get the horse re-shod. But where was the nearest blacksmith?

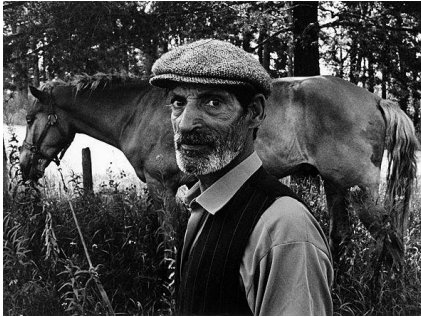
Alison remembered that just before Picket's Lock they had passed a gypsy camp. Now gypsies know a lot about horses, and usually made their own horse shoes, so they thought it was just possible that one of them could be persuaded to help.

They moored the boat and left Galois in charge of it while the others led the horse back to where the gypsies were camped. They went into the first caravan they came to and a rather large lady with a scarf tied around her head said, "of course my dears, we can help you. My brother looks after all our horses and I'm sure, for a small price, he will have your horse re-shod in no time."

She led them to the next caravan and a dark-faced gypsy came out. He agreed to help and said that for one

pound he would replace the lost shoe. His sister then said, “while he’s working on it why don’t you all come back to my caravan and I’ll make us a lovely cuppa – and I’ll tell your fortune for free.”

So they went back and, over tea, she told their fortunes. Alison was told that she would soon go on a great journey.



“Not a bad guess,” thought Alison. “My Australian accent is a dead giveaway that I’m likely to go back there before long.” In fact father had almost completed his sabbatical year at the University and they would be returning in a couple of months.”

The gypsy woman continued, “in a few years time, you’ll fall in love with a tall dark young man called Charles and one day you’ll access the inaccessible.”

“I’d hardly call Brother Charles young,” she thought. “And he’s rather short. And I love him dearly, but I wouldn’t say I was in love with him. But what’s that about accessing the inaccessible. Sounds like something father would say.”

The others had their fortunes told. Emily would live and work in a sandy valley. And Brother Charles would rise to a top job in his profession. This all sounded very vague. Why didn’t she come out and say that Brother Charles would one day become Pope!

At first the gypsy didn’t want to tell Galois’ fortune. I suppose she was unsure about predicting the future of a beaver. How could she find the lines of fate on a beaver’s

paw? But eventually, she said “One day you’ll have as many descendents as there are stars in the sky ... and they’ll all live in Scotland.” Emily recalled that something similar had been said to Father Abraham but the knowledge of the universe had improved tremendously since then and even a gypsy should know that there are billions upon billions of stars in the universe. If that many beavers settled in Scotland, their weight would cause it to sink beneath the sea!

Still, it was an entertaining afternoon. Finally her brother came in and announced that the horse was shod, and they could all go on their way. But as they were leaving the gypsy woman brought out a silver charm bracelet. She went up to Alison and put it on her wrist. “Only a pound,” she said:

Alison looked closely at the bracelet. It had about a dozen gold charms attached to it. One was a gypsy caravan, and one was a horse. Another was a narrow-boat, and there was a lock key and Buckby Can – a bucket for drinking water on a narrow boat.

Alison was very impressed with the bracelet. “It’s lovely. The only thing is that it’s a pity that the water can is all silver instead of being brightly painted like a real one. But I love the narrow-boat. Look, the tiller at the back even moves.”

There was a duck (there were plenty of those along the canal) and a sheep (they had seen many). It would make a real remembrance of their journey. But Alison had wandered into this strange world without any money and had to rely on the generosity of others. Besides, if she had some modern money the gypsy probably wouldn’t accept it.

However Brother Charles was as generous as he always was. "Alison, if you would like it, you may have it. It would be a worthy addition to your collection of axioms."

"But is it really an axiom? Which one is it?"

"Why the Axiom of Unions. Each charm is a separate piece of jewellery, but they are all united in this bracelet. And remember, the charms on a charm bracelet are supposed to bring good luck. Who knows you might meet your tall, dark handsome man one day."

"But she didn't say anything about handsome."

Charles was usually very modest, but on this occasion he couldn't resist saying, "If his name is Charles, he's bound to be handsome!"

"And one day you might be Pope."

"I'll settle for Bishop."

Emily was a bit dubious about the prediction that she would live and work in a sandy valley. That didn't seem to be all that appealing. And Galois was musing about his uncountably many descendents. At least Scotland would be a nice place to live."

So Brother Charles handed over two pounds, one for the charm bracelet and one for the horseshoe, and they went out to where the newly shod horse was standing. They were about to lead him away when Alison tugged the arm of Brother Charles and said, "don't you think there's something strange about the horse? He's got the same colouring but he looks so much older and tired than before."

"I guess these last few days have taken their toll on him. He's had a lot of work to do, pulling the boat."

"No, but are you really sure he's the same horse?"

“Well which horse do you think he is?”

“I’d be happy if you checked his four hooves. One should be bright and new.”

Charles inspected all four hooves and was puzzled. “They all look quite worn. The gypsies have given us a second-hand horseshoe.”

“But the nails are all worn. Surely they would have used new nails at least. No, I don’t think this is the same horse as we’ve had all along.”

Charles confronted the Gypsy, but he just shrugged his shoulders. “It is possible that your horse wandered away after I had shod him, and this one might have come up in his place. I got called away at one point. If you can find your horse among all these then you can have him.” He pointed to where twenty or thirty horses were grazing in a far-off field.

The next hour was spent in going around from horse to horse, inspecting their shoes. It was hard to keep track of which ones they had seen and Emily was quite sure they had inspected some of them more than once. They didn’t just look at the ones that had the right colouring because it was well known that Gypsies could change the colouring in an instant.

Eventually they came across one that had one hoof shod in a bright new horseshoe. His colouring was different to theirs but Emily said she was sure it was the right one because she remembered that it had a distinctive mark on its right ear. “I remember it was a bit like the number three.”

The gypsy didn’t dispute it, so they swapped the horses over and walked back to the boat. When Galois heard how they had been tricked he grabbed his sword. “I’ll cut off his ear, for such treachery!” It was all they could do to restrain

him. It was only when Brother Charles said that he was anxious to get back to Tottenham that Galois relented. "He'd better look out the next time I'm in this area."

On their way back Galois told them a story about his homeland. He called it the Great Busy Beaver Problem.

"You see we 'ave ze great sportin' event in my country. It's ze Beaver Endurance Competition. P'raps you know zat we beavers are verry busy. Well, we 'ave beavers from all ze provinces come to compete. Zese beavers 'ave to sweem back and forwards across a stream. Ze winner is ze one who crosses ze stream ze most number of times. But we have ze leetle problem. How do we know who is ze winner?"

"That doesn't seem very difficult to me," said Alison. "You just wait till the last one stops."

"But some of zem can go on forever. However the rules of the competition disqualifies zese. It is ze one who goes the longest, but still 'alts at ze last, who is ze winner."

"But surely everyone will have to stop eventually. It's just a matter of waiting."

"Oh, you don't know ze stamina of some of zese beavers. Some are capable of going on forever. Last year we 'ad Elise – she stopped after she 'ad done *deux cents, soixtane sept* crossings. Zat left Norman and Denton still going. After zay had each done *un mille* crossings ze committee conferred."

"Well, clearly Norman and Denton were going to go forever, so the prize should have gone to Elise."

“But ’ow did we know? P’raps Norman would ’ave stopped after a few more crossing. Zen ’e would ’ave been ze winner.”

“And if Denton stopped a little later then he would have won. I see your problem. But surely the doctors could have examined all the contestants and eliminated those that had the constitution for going on forever.”

“We zought of zat but zis was another problem – ze ’alting Problem. We could find no way of telling which beavers would stop and which would go on forever.”

“Well, medical science is improving all the time. Perhaps one day your Halting Problem will be solved.”

“I doubt zat, but I can’t prove it.”

And so, this slice of the adventure was almost at an end. They stayed overnight at Tottenham and were met there the next morning by Ivy in her cart. “Do you want to check the hooves to make sure it’s the same horse that brought us here?” Charles asked Emily.

She replied that it wasn’t necessary and, in any case it was clearly a different horse. This one was several days older than the one that had brought them.

TO CHOOSE OR NOT TO CHOOSE

They woke up the next day to beautiful sunshine. Although Spring was not very far advanced, one could feel the warmth flowing in the windows as soon as they were opened. The blossoms in the Monastery gardens were sending scented messages to the bees that they were open for business and the birds were overjoyed at it all.

At breakfast Ivy announced, "well what a loovely day for the fair! I hope as ye'll all be a'goin'."

"What's zis zing you call a fair? We don't have zuch a zing in my country," said Galois.

"It's where a lot of people set up tents and serve cake and lemonade and you can win prizes for knocking down wooden blocks with balls or go for a ride on a steam merry-go-round," said Alison.

"And even have your fortune told," said Emily. "I wonder whether you'll get a more accurate estimate of the number of your descendents. As many as there are stars in the sky was a little exaggerated."

"Perhaps this fortune-teller will be a little more specific about my future prospects," murmured Brother Charles as he helped himself to more bacon.

"If there is a fortune-teller it won't be a real Gypsy," said Ivy as she started to clear the table. "It'll only be sumun from the Women's Inst'ute. They're raisin' siller for fixin' the bells in St Josephs. The tenor's done been cracked and it sounds sumpin' awful."

"Perhaps there'll be a magic show in one of the tents," said Brother Charles. I remember going to a fair when I was a

boy and there was this magician chap who did amazing things with metal rings.”

Well, it was too nice a day to spend indoors and so they all decided to go to the fair. After she had washed up the breakfast things Ivy hitched up two horses to the special carriage that was used only for visiting Bishops, and they set off.

The fair was held a little way from the village, in a large field belonging to Farmer Giles. There were brightly coloured tents and pavilions and notices promising all sorts of entertainment. Ivy was especially excited. She had brought a cake to enter into the cake-decorating competition. It was sure to win first prize, and it did. It was a replica of the main building of St Huberts, with turrets and iced windows and a flagpole. Even the flag of St Hubert was edible – made out of some sort of rice paper and decorated with cochineal and other colourings.

But she didn't linger among the domestic entertainments – the patchwork competition and the knitting exhibits. She wanted adventure. There was a giant slide – not quite so substantially built as the ones that Alison had slid down at the Royal Easter Show in Sydney. It was made out of timber and used a large tree for most of its support. You had to climb a ladder to get to the top and the timber slide was only wide enough for one person at a time. But it had something that was never seen in the 21st century world. Half-way down it had a curve and then it disappeared over a hedge, so that you couldn't see what happened at the bottom.

The attendant gave Ivy a mat at the bottom of the ladder and said, “lady, are you sure you want to wear those clothes?”

“What’s wrong with these?” She had a bright red skirt and lace top and was wearing her very best hat. She looked very smart. So she climbed the ladder. “It’s your choice,” the young lad called after her as she disappeared up among the branches.

What the boy should have explained is that after the slide disappeared around the corner and over the hedge it deposited whoever was sliding down into a tank full of water. There was no mattress to break your fall. They relied on the water.

The slide was only meant for children, who wouldn’t mind a soaking. The young attendant didn’t know how to say this to an old lady.

Splash!! Ivy got a real dunking. When she came up for air she was screaming ... screaming with laughter. “Good heavens, I haven’t had so much fun since I was a little girl.”

Fortunately Mrs Giles, the farmer’s wife was able to supply her with dry clothes. The hat, however, was completely ruined and so Ivy donated it to one of the horses to wear on the return journey.

Alison and Emily went to the tent where there was a prize for guessing the weight of a young pig. Emily reached into her bag and brought out a tape measure. Now Emily’s handbag is renowned in the world she came from. Although it wasn’t very big it contained every conceivable thing one might need in any circumstance. As well as a tape measure

there was a pencil and notepad, a couple of safety pins, a compass, a tiny pair of scissors and even a pair of artery forceps. The forceps were a gift from her mother, who was a nurse. Emily had not yet had occasion to use them for arteries, but you'd be surprised how useful they can be in holding fiddly bits and pieces when you're trying to make some delicate operation.

She measured the length of the piglet, from head to tail, and then the distance round its tummy. It was 30 centimetres long and 48 centimetres round. "If she was a cylinder," said Emily, "she would have a volume of 5500 cubic centimetres. Allowing for her pig shape we should take 80 per cent of this, say 4400 cubic centimetres. Now the specific gravity of young pig is 1.04, which means that her weight would be 4.576 grams ... let's say 4.6 kilograms." Emily did all these calculations in her head.

"Mature pigs can have a specific gravity of 0.9, which means that they can float in water. That's because they have a higher percentage of body fat."

So, going up to the man in charge of the 'guess this weight' stand, she said, "I estimate a weight of four point six kilograms."

"None of your Frenchy weights here – this is England."

"Sorry, well that would be six pounds, two and a half ounces."

"It's one shilling per guess. The weighing will be done at four o'clock this afternoon. You can have *three* guesses for half a crown. "

“Why would I need three guesses? They’d all be the same because I’ve given you the exact weight – to the nearest half ounce.”

When she wanted to be, Emily could be really sure of herself. As much as Alison would have liked her to win the prize she secretly hoped that the guess was wrong.

Alison went off to the horse-riding. They had pony rides for the children, where they were led around in a circle on a really tame old pony. But Farmer Giles had a proper horse on which he was allowing people to have a half-hour ride for ten shillings. “Just don’t go further than the village and mind that you’re back in thirty minutes.” So Alison paid her money and went off at a gallop.

She didn’t have a helmet, but they didn’t seem to worry about that sort of thing in the 1930’s. And she was lucky to have worn trousers. Now in the 1930’s it was rather unusual for a well-bred young girl to wear trousers but Alison had insisted on borrowing a pair from the gardener, who fortunately was about her height. There was no way she wanted to go to a fair in a skirt – not after what happened at Luna Park in Sydney.

There was one of those Coney Island attractions where you walked over a grid and the air blowing from underneath blew your skirt around your ears. It was so embarrassing. So just in case they had one of those she begged Larry, the gardener’s assistant, to lend her some trousers.

It was exhilarating riding at full gallop, across the fields towards the village. Probably it was just as dangerous as Ivy’s driving, but at least *she* was in control and she was not afraid.

Meanwhile Galois was taking part in the re-enactment of some local battle. He was annoyed that they didn't let him use his nice bright sword that he always carried with him. A wooden sword, painted silver, is not quite the same thing. Still he enjoyed fighting the Normans as much as if he was an Englishman.

Brother Charles amused himself throwing balls at stacks of blocks and won a couple of prizes. Then he met up with Emily and they browsed among the second-hand books. Ivy, in her borrowed clothes, went to the tea tent and caught up with some of her friends.

Once Alison returned, unscathed, they rounded each other up and went to the lunch tent where they were serving cucumber sandwiches and elderberry wine.

"There's so much to choose from," said Alison, "this is one of the best fairs I've ever been to ... in my whole life". She narrowly avoided ending the sentence with a preposition.

"What shall we do after lunch?" asked Brother Charles.

"Well there's a tractor race, and a tug-o-war," said Ivy. "And there's this mind-reader Gypsy woman."

"Why aren't you helping," Emily asked Ivy. "Aren't you a member of the Women's Institute?"

"Well they wouldn't let me do the cake judging."

"That's because they knew you were going to enter it."

"Well, what else could I do? Run the tug-o-war?"

It was no good arguing with her. Probably the real reason was that she would rather wander around having fun than be stuck in one place doing some job or other. I think, in a way, she had never grown up. She was a bit like Peter Pan, though she was carrying rather more weight than he did and

would never have been able to get off the ground if Tinkerbelle had sprinkled her magic powder.

After the tractors and the tug-o-war they all made for the Gypsy Mind Reader's Tent. They went inside and at first they could hardly see. There was a small candle that gave a bit of illumination. The Mind Reader was dressed in a bright red dress, a shawl and a scarf, with a rather ugly masculine face. In fact he looked rather like Mrs Jones' husband dressed in his wife's clothes, but it was too dark to be sure.

Alison reflected on the fact that for something put on by the Women's Institute there were a lot of men. Probably the women were concentrating on the catering and the handicrafts. Everything else seems to have been run by their husbands.

He sat behind a table covered in a dark blue cloth, edged in gold. On the table was a sort of magician's hat and a magic wand. Mr Jones, if that's who he was, took a die from the hat and passed it around for us all to examine. It was a little bigger than the plastic ones you use to play Snakes and Ladders, and it was made of brass.

The dots, instead of being painted, were small diamonds. Well, I don't expect they were *real* diamonds, but they sparkled in the candle light. The 'one' which is usually a bigger dot than the rest, was a large 'diamond'. I expect you've already worked out that there were 21 diamonds in all.

"Now I want you to *choose* one of the six numbers and hold the dice in your hands while you concentrate." He emphasised the word 'choose'.

Of course, being only one he should have called it a 'die' – it's only dice when there is more than one. I expect that you knew that but Mr Jones, who was only a farmer, didn't.

Alison was the first to volunteer. She took the brass cube in her hand and closed her eyes for a moment. "Now give it back to me." Mr Jones took it and put it into the hat.

"Now tell me what number you chose."

Alison said "five."

He then waved his wand over the hat and then took out the die and gave it to Alison. "Now concentrate on the number five and roll the die." Alison closed her eyes again and threw the die so hard it rolled onto the floor. Galois dived under the table and said, "my goodness, *c'est un cinq*." It was indeed a five. He gave the die back to Mr Jones who put it back in the hat.

Emily wanted to try it. Mr Jones gave her the die and she immediately said "one". Again the die was returned to the hat, while abracadabra was said over it. Then it was returned to her and she rolled it very delicately so that this time it stayed on the table. They all looked, and indeed it was 'one'.

Ivy chose 'two' and lo, it was a 'two'. Brother Charles chose 'three', for the Trinity he said, and it came up 'three'.

"What's this got to do with mind reading," asked Alison. "You're not reading anybody's mind."

"Ah, but the dice is." He had been speaking in a high pitched voice so that he sounded like a woman.

"That will cost you two shillings. It's sixpence a go."

Brother Charles paid the money and they started to go out. 'I think I know how he does it,' Emily muttered to the

others. Then she stopped and asked if she could have another go. “By all means, have you got another sixpence?” Charles handed over the money and this time, when Emily held the die, she frowned as if she was concentrating really hard.

“Have you chosen a number?”

“Two is a rather nice number.”

They went through the ritual of blessing the die in the hat and when it was returned Emily threw it. It came up ‘four’!

“Close,” said Alison, “but not quite right.” Mr Jones looked a bit uncomfortable. “I’m sorry I don’t know what went wrong. I’ll give you back your sixpence, that’s fair.”

But Emily had a strange look on her face as if she’d seen a ghost. “Keep your sixpence mister. I said ‘two is a nice number’ but I didn’t say that it was what I had chosen. I *had*, in fact, chosen ‘four’!”

At this there was a loud squeaking noise coming from outside the tent, and a lot of shouting. The next moment a piglet – the piglet, whose weight had to be guessed – came running in followed by a small crowd, trying to catch it. In all the confusion the table and the magician’s hat was overturned. After about ten seconds the piglet slipped out under the side of the tent and the small crowd ran out the door.

They picked up the table, and the cloth, and the hat and the magic wand. But they couldn’t find the brass die with its 21 diamonds. The fact that



it was so dark didn't make the job of finding it easy, but at the end of ten minutes they were all convinced that it had gone.

"Perhaps that infernal pig has swallowed it!" said Mr Jones in his normal voice, "if they catch him I'll make them cut him open!"

"But what if he didn't swallow it? It wouldn't be fair to cut him open just in case."

"No need to cut him open. They must have weighed him before the guessing began. If they weigh him again and he's an ounce heavier he's sure to have swallowed it. Otherwise ..."

"But he must have lost some weight with all this running around."

Our four friends went off to the lemonade tent and sat down to compare notes. "How d'ye figure he done it?" asked Ivy.

"Well, I thought he might have had six dice tucked away in his hat, each one looking the same. I'm sure he had them tucked into special pockets, in order, so that he could reach in and pick out the one we chose."

"But how did that make the die come up with the right number?"

"Well, perhaps each one was weighted so that one particular number would be sure to come out on top. The 'three' dice would always come up 'three'. So if you said 'three' he put the die you were holding in the hat and secretly swapped it for the die that would always come up 'three'.

“So that’s how he did it,” said Alison, “but when you chose ‘four’, but said ‘two’, why didn’t he pick out the die that always comes up ‘two’? Did he mess things up and select the wrong die?”

“I suppose so, but it’s strange that he inadvertently chose the same number as me. What a pity it got lost. I would love to try it out some more.”

Galois grinned, “who said anything about lost?” He held up a brass cube with 21 gleaming diamonds.

“Where did you find that?”

“Oh, it was on ze table at zat moment ze pig came in. I had my `eye on it and as soon as ze table was knocked over I picked it up and, ahem, secreted it about my person.”

“You mean you stole it?”

“Well, the man was obviously a charlatan. So he deserved to lose his die. You know, this could be another ‘axiom’ for your collection. You could put it on a chain and wear it round your neck.”

“I could never do such a thing,” protested Alison. “You have to give it back.”

“The Axiom of Choice,” announced Brother Charles. “I agree that you have to give it back, but what a worthy addition it would have been to your collection.”

“And what is the Axiom of Choice?”

“It is the assumption that if you have a whole lot of boxes, none of them empty, you can choose one thing out of each box.”

“Well I don’t call that an axahem,” said Ivy. “It’s the gospel truth. No-one can stop me choosing one thing from each box.”

“Indeed it *is* clearly true if there are only *finitely* many boxes and only *finitely* many things in each box. The Axiom of Choice is relevant to *infinite* mathematics. There it’s only an assumption.”

“So one can never prove that it’s true?”

“That’s right.”

“But how can you be sure of that? How can you know what someone might prove in the future? After all they said that man would never walk on the moon, and look what happened,” said Emily. Brother Charles looked puzzled. “What has happened? Oh, I see: you mean the man in the moon.” Emily then realised that, of course, they were in the 1930’s and knew nothing about what took place in 1969.

“I know that nobody ever will prove it true because, about ten years ago a friend of mine, Abraham Fraenkel *proved* that it is logically impossible to prove the Axiom of Choice.”

“Doesn’t that make it false?”

“No, it’s perfectly possible that one day someone will prove that you can’t prove it false. Then it will be completely undecidable.”

Emily said, “it’s coming back to me. I remember Dad talking about something like that and he said someone had proved that you can believe in the Axiom of Choice, or believe that it is false, and no one can possibly prove you wrong!”

“So it is like a creed,” said Brother Charles. “Just like nobody can prove that God exists and nobody can prove that He doesn’t. That’s what faith means – believing in something that is undecidable logically, but which makes a more

satisfactory view of the world. There are several undecidable things in mathematics. Logic is great but it can only go so far.”

“So we should be an agnostic – if we can’t prove it either way?”

“Well there may be other reasons for believing than mere proof. I believe in God because it helps me to make sense of the world. I believe in the Axiom of Choice because it leads to better theorems.”

Alison was getting worried. “But mightn’t bridges fall down if engineers make the wrong choice?”

“No, anything of a practical nature will be true or false irrespective of whether or not you accept the Axiom of Choice. For all practical purposes, in this finite world, it doesn’t matter. But theorems based on the Axiom of Choice are simpler to state than without it. That’s why most mathematicians are happy to assume it.”

“What on earth is you people talkin’ about,” said Ivy. “I don’t need to have an axxything to prove that I can choose. I just does it.”

“That’s because you believe you have free will. There are some who say that we’re all just machines, and our thoughts are controlled by our biochemistry, and that free will is just an illusion.”

“Perhaps one day scientists will prove that that is so,” said Alison.

“But if materialism is true, there’s no validity in logic and so such a proof would be like fairy floss. And if materialism is not true we probably can’t prove that either.

With certain things it comes down to believing what suits you.”

By now, at the other end of the table, Ivy and Galois were doing some more practical research. They were rolling the die. They had decided a long way back that all this talk of proof and undecidability was not only incomprehensible, but it was downright boring! Perhaps you felt the same and you’ve skipped over the last couple of pages.

“I must say,” said Galois once the others stopped their philosophical discussion and looked at what they were doing, “this die is rather remarkable. I choose a number and roll the die and about a quarter of the time it matches my choice, even though I choose different things each time. By rights it should only match my choice one time in six. I rolled it 24 times and it agreed with my choice six times instead of the expected four.”

“That might be just by chance. The next 24 times it might only get it right twice.”

Alison offered the following alternative explanation. “It could be that your subconscious influences the way you roll the die. Emily, you remember we used to hold a string attached to a weight and we’d close our eyes and someone would ask a question and we’d concentrate on the answer – yes or no. If the answer was ‘yes’ the weight would swing in a straight line and if it was ‘no’ it would go round in a circle – or was it the other way round, I can’t remember.”

“All this is very interesting,” said Brother Charles. “But what is more important at the moment is to restore that jewelled cube to its rightful owner. I think, Mrs Dunn, you

said that it was Mr Jones. I expect he's already packed up and gone home so we'll have to ask the vicar for his address."

When they located Mr Jones he said that he wasn't going to do the mind reading act ever again and so Alison could keep the jewelled die. Alison had yet another axiom to add to her collection – the Axiom of Choice.

RAMANUJAN'S RING AT LAST

They were still awaiting permission to drain the canal at Waltham Abbey when they were contacted by the Kent County police. The criminals who were arrested had admitted that they had stolen a ring that answered the description of the Ramanujan Ring and that they had put it in a bag of other jewellery and sunk it in Holehaver Creek. They gave a full description of where it could be found.

“Why, that’s not that far from here,” said Brother Charles. “It’s certainly a lot closer than Dungeness!”

Galois was keen to set off that very moment. “How do we get there?”

“Well, we’ll get Mrs Dunn to takes us to West Thurrock. From there we can hire a small boat that will take us along the Thames, past Tilbury, and up Holehaver Creek. We’ll set off first thing in the morning.”

The next morning they set off in the cart, with Mrs Dunn taking the reins. In just over an hour they had reached West Thurrock, on the Thames, and hired a rowing boat. Mrs Dunn then returned back to St Huberts while the other four rowed up towards Tilbury. It took them most of the day to row up to the mouth of Holehaver Creek. As it was getting late they decided to camp on Canvey Island. Luckily they had prepared for this and had brought a small tent. Alison and Emily slept in the tent and Charles and Galois kept watch outside. But before they turned in they had a meal on a campfire. Galois had caught three fish and offered to cook

them. While they were eating, he went back to a nearby tree and stripped off some bark. “Delicious!” he said.

There is no doubt that food cooked out of doors tastes much nicer than food prepared in a kitchen. They all fell asleep happy, with full stomachs.

The next morning they went to find the place where the thieves claimed to have stowed the stolen items. It was on a small creek, where the water comes tumbling down a hill and into the main creek. Their clue was a certain tree, with a distinctive shape, on the edge of the creek. They searched up and down this section of creek but could find no such tree. Either the thieves were lying, or the tree had been removed.

“Well that settles it – we’ll have to drain the creek,” announced Galois.

“What *all* of it?” asked Emily.

Alison couldn’t understand. “Not the whole of Holehaver Creek? That would be impossible because it’s tidal. We’d have to empty the entire English Channel!”

“No,” laughed Galois. “If it’s here I’m pretty sure it’s at the bottom of this small creek that runs into the main one. If we were to dam it upstream, pretty soon it would empty.”

“But how are we going to do that?”

“Leave it to me.”

So Galois went upstream and chose a spot near where there were a lot of largish trees. Then he set to work to chew at the base of one of them until it toppled into the water, across the creek. You see, this is what beavers are good at – building dams.”

It took several hours and, as the sun set, the job was only half done. They returned to the main creek where they had left their boat. By now it had started to rain. So while Alison and Emily slept in the tent Charles and Galois dragged the boat onto the shore and turned it on its side. This would provide some protection for them during the night.

The next morning the rain had stopped and Galois resumed his engineering works. After five trees had been felled, and were lying across the creek, Galois fetched some smaller twigs and leaves to plug up the gaps. If you or I had built such a dam, the pressure of water building up behind it would have swept it away. But beavers know how to make a dam strong.

The water below the dam flowed downhill towards Holehaver Creek and after a while the creek bed was visible. But they could see no bag such as the thieves had described. But if there *had* been a bag it was probably half-buried in the mud.

They poked around in the mud, going up and down the creek. About an hour later Emily called out, “here it is!”



She pulled a leather bag, encrusted in mud, out of the creek. Galois took a small bucket that he had in the boat for bailing out water, and went up above the dam to fill it. He brought it back and poured

it over the bag, and cleaned it as best he could. They opened the bag and poured its contents onto the grass.

There was a wonderful collection of jewels, probably worth hundreds of pounds. But there was only one piece of jewellery they were looking for.

“Is this the ring?” asked Alison holding up a gold ring, set with a large ruby. They washed it and it began to sparkle in the sunshine.

“Put it on!” shouted Emily to Alison. See if it gives you the gift of mathematical insight.

Alison put on the ring and stared at it, as if in a trance. As she stared, her eyes started to cloud over and her face became white and clammy. Then she fainted.



When she came to, after a few seconds, Emily sat her up. Was there a strange power in this ring? “Alison, did anything mathematical come into your mind?”

“No, I just thought that the last person who wore this ring is now dead. It was a creepy feeling.”

“Did it give you any feeling of power?”

“Not really.”

Emily took the ring from her finger and put it on her own. She was pensive for a whole minute, and then she took it off. “This ring doesn’t have any special power. Are you sure that it’s Ramanujan’s ring?”

Galois inspected it closely. “It certainly fits the description down to the last detail. I’m sure it is.”

“Well it’s known that Ramanujan had tremendous mathematical insight, but it couldn’t have been from wearing this ring. He may have *believed* that it gave him intellectual power – I remember reading that he said his insight came from some Indian goddess. But the power must have come from within himself.”

Of course they would have to turn all this jewellery over to the police. When she first found the ring Emily had been tempted to not say anything about it and quietly put it in her pocket. But I’m sure, if it came to it, she wouldn’t have done anything so dishonest.

But the question never arose. Once she realised that it was just a piece of jewellery she lost all interest in it.

Alison, who loved jewellery, and might have treasured it simply as a beautiful ring, was never tempted to keep it because the thought of wearing a dead man’s ring made her shudder!

So they bundled up the jewels, back into the bag, and dragged the boat back into the water. Perhaps Galois should have gone back up the hill and broken open the dam, but the thought never occurred to him. When somebody discovered it a day or two later they said, “how strange – it looks like the sort of dam a beaver would make. But I don’t think there are any beavers in this country!”

And so the quest has ended and the Ring of Ramanujan was found. But it did not have to be destroyed in the Mountain of Doom. It was indeed a *nilpotent* ring, having no power of its own. Ramanujan's intuitive genius came from within himself.

READ THIS LAST

We now come to the part, in nearly every fantasy story, where Alison should be waking up to find that it had all been a dream. That was what happened with Alice.

So, we expect to read that Alison heard Emily's voice as if at the end of a tunnel.

"Come on dear Alison, I know I'm a bit late, but there's no need to have fallen asleep on that doorstep."

And Alison would realise that she had indeed fallen asleep on the doorstep of number 24 Leinster Gardens, and that only about twenty minutes had elapsed since she got there.

But perhaps what we call the *real* world is just a layer in a dream. Have you ever dreamt that you were dreaming? You woke and said to yourself, "oh, that was just a dream." Then you wake up properly and realise that you were still dreaming when you said that. But perhaps what we call the *real* world is just a dream and that when we die we wake into ultimate reality!

So it's time that I told you whether this story is meant to be true, and how it came to be written. You see *I* am Emily Christianson. Yes I know it says Emily Bronowski on the cover, but I'm married now to Jake Bronowski. By the way he's not related to the famous Jacob Bronowski who wrote *The Ascent of Man*. I must confess that Jake having a name so similar to Brontë was part of the attraction in the early days

of our courtship. Emily Bronowski sounds a bit like a Polish version of Emily Brontë.

My older sister is indeed Alison and as teenagers we had a series of strange dreams. What's more we kept barging into each *other's* dreams. Many of the incidents of this story were in one or other of our dreams.

We talked at great length about these incidents, and embellished them. When we were a little older Alison persuaded me to write them down. But it's not all fantasy – much of what you have read is really the truth.

Alison *did* get annoyed by jokes about her name, and I *am* really a fan of the Brontës. I didn't actually believe that I was the reincarnation of Emily Brontë, but it *is* true that I was pretentious enough, back then, to insist on putting the two dots over the 'E' in *Ēmily*. I was a bit silly back then and now I've reverted back to plain old 'Emily'.

There really *is* a house at 24 Leinster Gardens that's merely a façade. You can Google the address and find a picture of the façade. And we really *did* live in Leinster Gardens and walked past that façade countless times. The 'house' was already there back when J.M. Barrie, who lived at the end of the street, wrote *Peter Pan* and it's a wonder to me why he didn't incorporate it into his stories.

The façade was built some time after the Circle Line that runs underneath. Because there needed to be an open section to the line (to let the smoke from the steam locomotives escape) there would have been an unsightly gap in the row of terraces if No 24 hadn't been inserted – much like a bridge that a dentist inserts to replace a missing tooth.

The fact that the adventures began on the Underground is a reference to the fact that Alice's took place underground and in fact the original name for *Alice in Wonderland* was *Alice's Adventures Underground*.

It is true that our father is a professor of mathematics who was on sabbatical in London. That's where we got to know so much about mathematics from a very early age. He used to tell us stories about the magical things in mathematics and he used to say that "mathematicians are the great story tellers of the scientific world".

Everything about mathematics in the story is indeed true, from multiple infinite numbers to the Busy Beaver Problem. Galois was made a beaver so that he could tell the story of this problem. I was amazed to find, much later, that Lewis Carroll had included a beaver in his poem *The Hunting of the Snark*.

What about Mum? Well, mothers never figure much in fantasies. We *do* have a mother, and we love her dearly, but she's never been very good at mathematics so it didn't seem appropriate to put her into the story. She doesn't mind.

Mum was a midwife, at St Mary's Hospital in Paddington. That's where many of the royal babies were born, though she was never involved in any of those cases. But she *did* get to meet the royal obstetrician, Mr Pinker, though she never actually worked with him. At the hospital she became friends with the head cook. Ivy Dunn is indeed her real name and she is one of those salt-of-the-earth people you can't help loving.

Brother Charles is real. Well his name isn't Charles and he isn't a Brother. But he *did* look like a white rabbit, with his twitchy nose and whiskers. His name actually was John and we met him when we were taking Jesus down from the cross. I thought 'Charles' suited him much better.

The Jesus we took down from the cross was a rusty old iron crucifix that used to hang from a pole outside Christ Church, Lancaster Gate. One bracket had rusted through, and Jesus was hanging precariously, and posed a real danger to anyone walking on the footpath underneath.

Alison and I discovered the problem, and spoke to a man who was walking by. Yes, I know we're not supposed to talk to strangers except in stories. But I felt that being on a mission to save Jesus nothing bad would happen to us.

Charles (I'll continue to call him by his name in our story) fetched some tools and a ladder, and we managed to bring Jesus down safely. We became friends with Charles and it turned out that he knew Dad well. He was a physics professor at the same university.

Ramanujan was indeed real, as you can see if you Google him. He really *did* have an uncanny ability to know mathematical truths without being able to prove them. But, as far as I know, he didn't have a ring. He attributed his ability, to know mathematics intuitively, to some Indian god that he worshipped.

The ring itself – a blood red ruby – was in Mum's jewellery box and, from an early age, we fanaticised about it having special powers. Another coincidence is that I'm helping to develop a computer language called Ruby – quite a

powerful computer language. If the queen ever gets to read this and asks me to dedicate my next book to her, as happened with Queen Victoria and *Alice in Wonderland*, I'll dedicate *The Data Structures of the Ruby Programming Language* to her Majesty.

There was a real mathematician called Galois, but he lived a long time ago and he was French, not French Canadian. We rolled up what we knew of him with our small dog. We used to dress our dog up with a white collar and red coat and take him for a walk in Kensington Gardens.

The ventrillo cat was obviously inspired by the Cheshire cat, but also partly by our own cat Flopsy. Flopsy was not able to disappear, leaving just her eyes visible, but she could make her eyes disappear, leaving everything else visible.

Our aunty lived at Upminster, so we knew the area pretty well, and it seemed appropriate to put St Hubert's there. And St Hubert really was the patron saint of mathematics. You see there's more truth in our story than fiction! Alison and I did ring bells, though not at Christ Church, Lancaster Gate.

Where we *did* stretch the truth a bit was making out that I knew so much more mathematics than Alison. I had a lot of trouble persuading her that the contrast would be better for the story. But I have to admit she is really quite smart – smarter than me. She did her PhD in mathematics at Queen Mary College. Her thesis was on logic, on inaccessible

ordinals – the gypsy was indeed correct! I graduated from Imperial College in computer science.

I am now doing software development in Silicon Valley. The weird thing is that at the time we put in that bit about the Gypsy predicting that I would live and work in a sandy valley I don't think I'd even heard of Silicon Valley.

Alison is now back in Australia (that's where we come from) lecturing in mathematics at the University of Wollongong, one of the smaller universities in Australia. She lives at Stanwell Park, where there is a really wonderful unspoilt beach and she commutes to Wollongong. I really

love being there whenever I visit her.

She has lots of friends and she loves surfing. She also goes hang gliding from the nearby Bald Hill, right down to the beach and she lives in that house on the headland between the two lagoons that you always see on the iconic photographs of Stanwell Park.



She has had some strange adventures there, and I'm trying to persuade her to write a book about them. But she says that there are far too

many detective stories around and nobody would be interested in one about a detective who's a mathematics lecturer. But I think they would be.

But Alison's very stubborn. She says that firstly, she *won't* write the book, and secondly, that *if* she does, it will be called *The Mystery of the Truncated Lecture*. Oh, well, some books have the strangest of titles.

Let me have the last word to any children who may have made it to the end. Look, you're not supposed to be here. I'd forgotten that this started out as a children's story. No, don't go away, but if I'd remembered there might be children still reading I wouldn't have made references to headless corpses. Just don't say anything to your mum.

I guess the reader I had in mind was the grown up who can still think like a child. When Lewis Carroll first told the story of Alice he had only the Liddell children in mind. They were in a boat on the river and he was entertaining them. But when he wrote it up to become a book he couldn't stop himself adding bits that only adults would be interested in – such as logic and the origins of words. Children still enjoyed it but they just skipped these bits. Then a man by the name of Martin Gardener wrote *The Annotated Alice* where he explained all the obscure references and jokes that Carroll made.

I now have four children – Alison has had lots of boyfriends, but she's still looking. Let me introduce you to them – I mean my children, not Alison's boyfriends. There's Alison – she's obviously named after my sister. Then there's

Johanna, the eldest – her name reflects our Scandinavian heritage. Next there's Danielle. She's only met Ivy Dunn once but she's become inspired to become a cook. And she wants me to point out that she's much so more refined than Ivy! And David – he might become part of the next generation of mathematicians.

When I showed the manuscript of this book to them they said, “mum it's a great story but ... there's too much maths.” That's probably true. If you followed every detail, which you probably didn't, you would have digested at least six complete mathematical proofs! That's more than in most maths textbooks these days, where they just tell you the rules and think you wouldn't be interested in the reasons why.